Lecture 7

$$d\mathcal{E} = dq + dw \underset{\text{usually}}{=} C\,dT - P\,dV$$

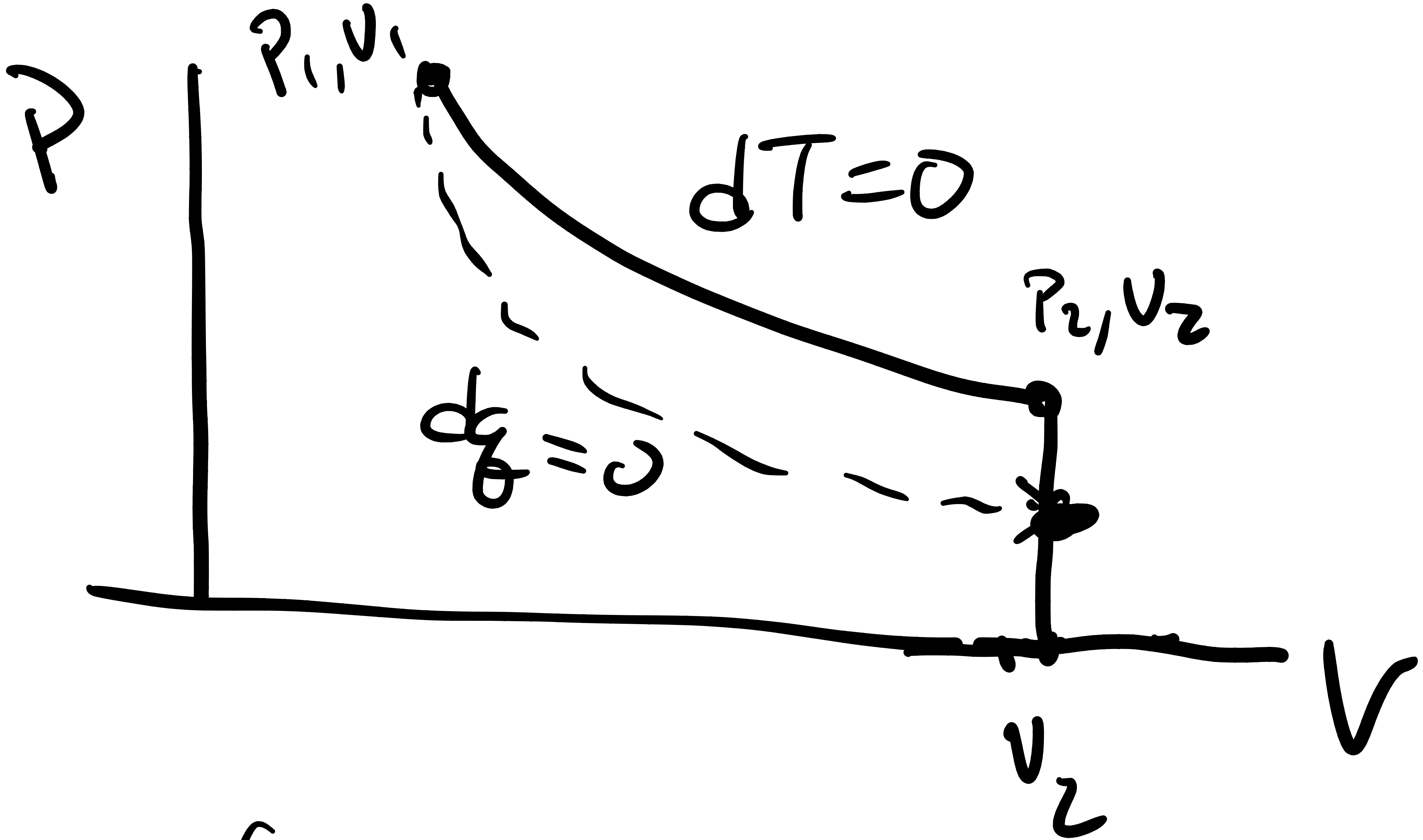
Covered 4 kinds of expansions - changes in state

① Const V

② Const P

③ Isothermal

④ adiabatic, $dq = 0$



(monoatomic)
for an ideal gas

$$\mathcal{E} = \frac{3}{2} nRT$$

$$PV = nRT$$

If we have a state function, $\mathcal{E}$

$$\Delta \mathcal{E}_{1 \to 2} + \Delta \mathcal{E}_{2 \to 3} = \Delta \mathcal{E}_{1 \to 3}$$

$$\Delta \mathcal{E}_{1\to 3} = \cancel{q}^{\,0} + w = -\int_{\text{path } 1\to 3} P\,dV$$

$\uparrow$ adiabatic

$$\int_1^2 \frac{1}{x}\,dx$$

$$\ln x \Big|_1^2$$

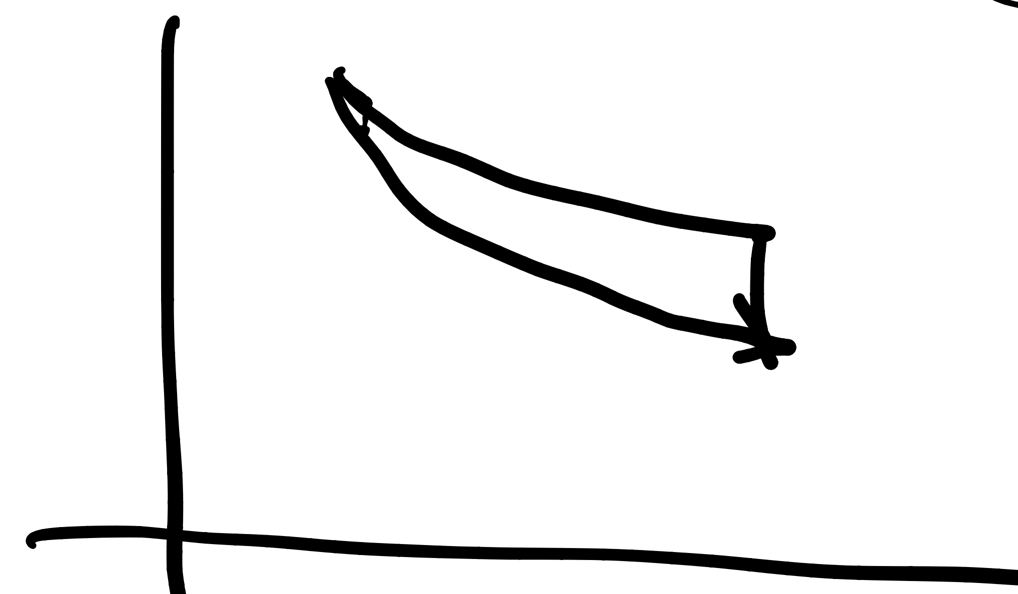
$$= \ln x_2 - \ln x_1$$

$$= \ln\left(\frac{x_2}{x_1}\right)$$

$$\Delta \mathcal{E}_{1\to 2} = 0 \quad \text{for an ideal gas}$$

$\uparrow$ isothermal

$$\mathcal{E} = \frac{3}{2} nRT$$



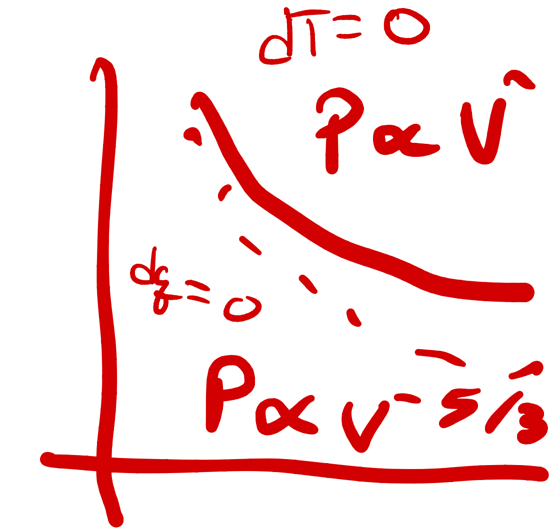
$$\Delta \mathcal{E}_{2\to 3} = C_V \Delta T$$

$$P \to \frac{nRT}{V}$$

$$\underbrace{-P\,dV}_{1\to 3} = \underbrace{C_V\,dT}_{2\to 3} \Rightarrow \boxed{-\frac{nR}{V}\,dV = \frac{C_V}{T}\,dT}$$

$\to$ integrate

$$-nR \ln\left(\frac{V_f}{V_i}\right) = C_v \ln\left(T_f / T_i\right)$$



final and initial temps
in adiabatic expansion

$$V_f / V_i = \left(\frac{T_f}{T_i}\right)^{-C_v / nR}$$

C_v – monatomic
$\frac{3}{2} nR$

$$T = \frac{PV}{nR}$$

$nR + C_v = C_p$

$\Longrightarrow$
Solve for P

$$P_f / P_i = \left(\frac{V_i}{V_f}\right)^{\frac{nR + C_v}{C_v}}$$

$$\gamma = C_p / C_v = \frac{(5/2)}{(3/2)} = 5/3$$

$$P_f/P_i = \left(\frac{V_i}{V_f}\right)^{\gamma} = \left(\frac{V_f}{V_i}\right)^{-\gamma}$$

$\hookrightarrow$ $$P = \lambda V^{-\gamma}$$

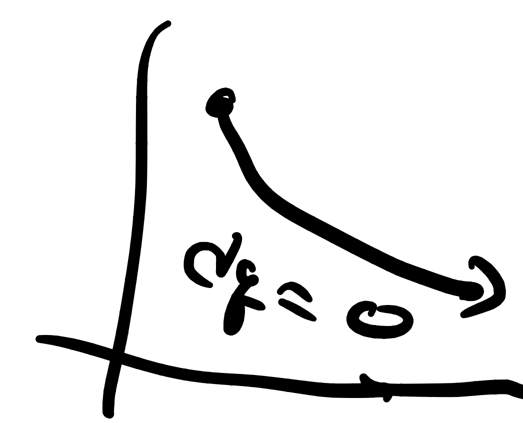
diatomic

$C_v = \frac{5}{2} nR$

$C_p = C_v + nR$

$$W = -\int_i^f P\,dV = \frac{+\lambda}{\gamma - 1}\left(\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right)$$

$$\lambda = P_i V_i^{\gamma}$$

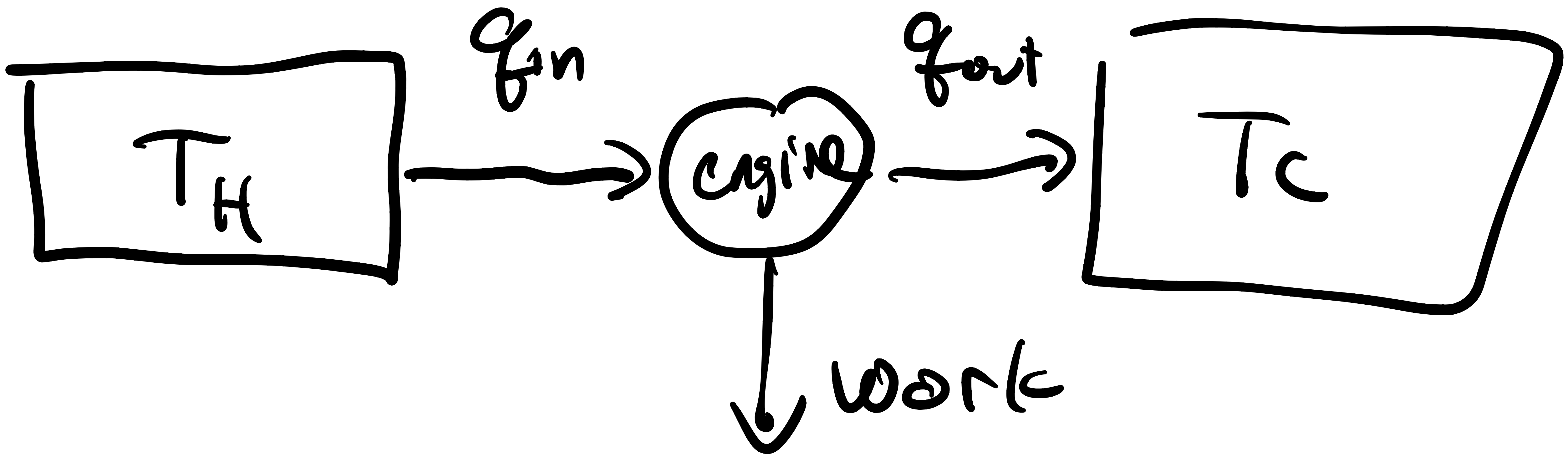


also

$\Longrightarrow$ $$W = C_v \Delta T$$ $\leftarrow$ plug in

$PV = nRT$

Goal: get work from heat transfer

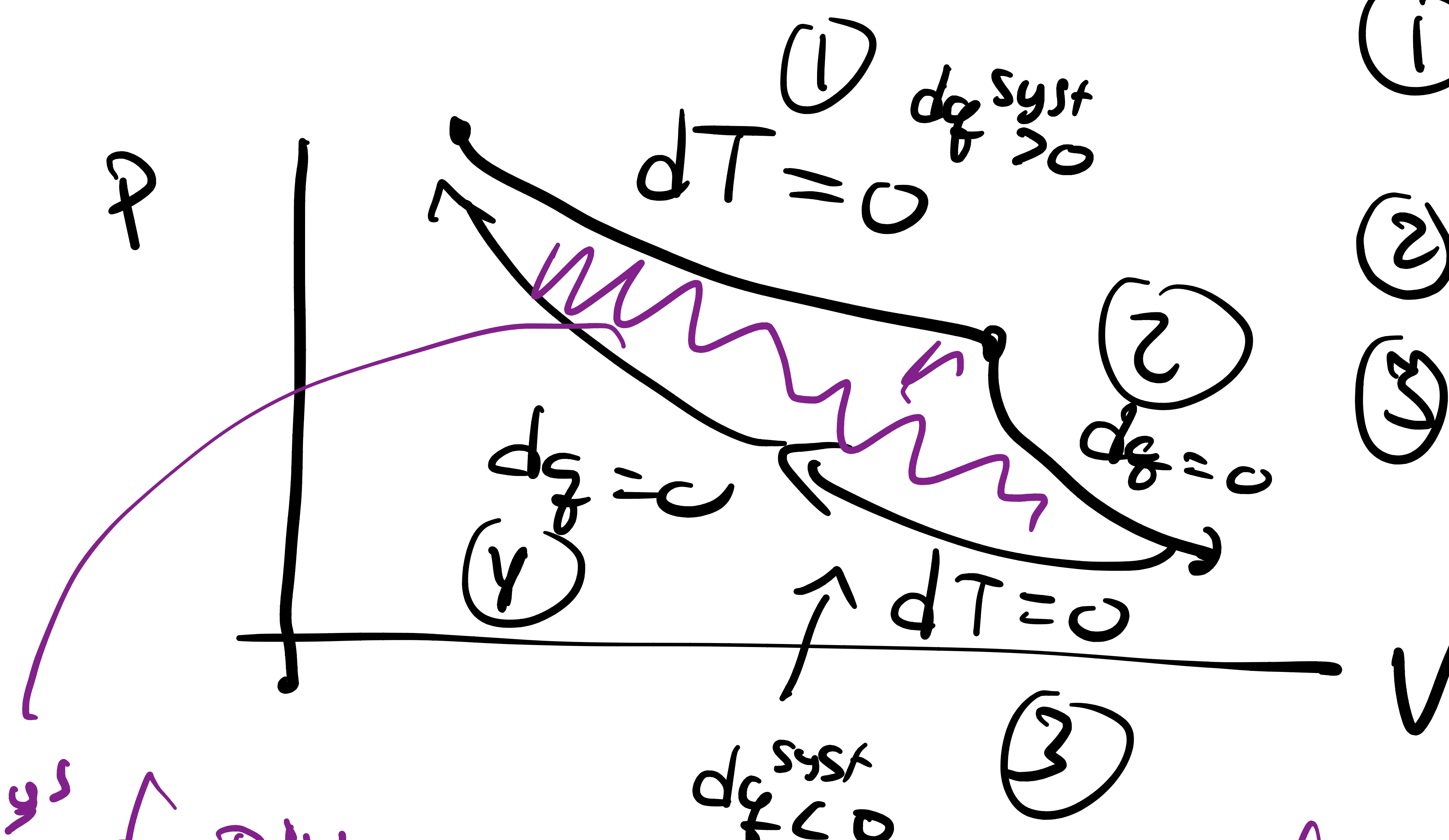


Energy is conserved, $q_{out} + \text{work} = q_{in}$

$$\text{Efficiency} = \frac{\text{Work done}}{q_{in}}$$

Carnot cycle (1824)

Engine has to complete a "cycle"

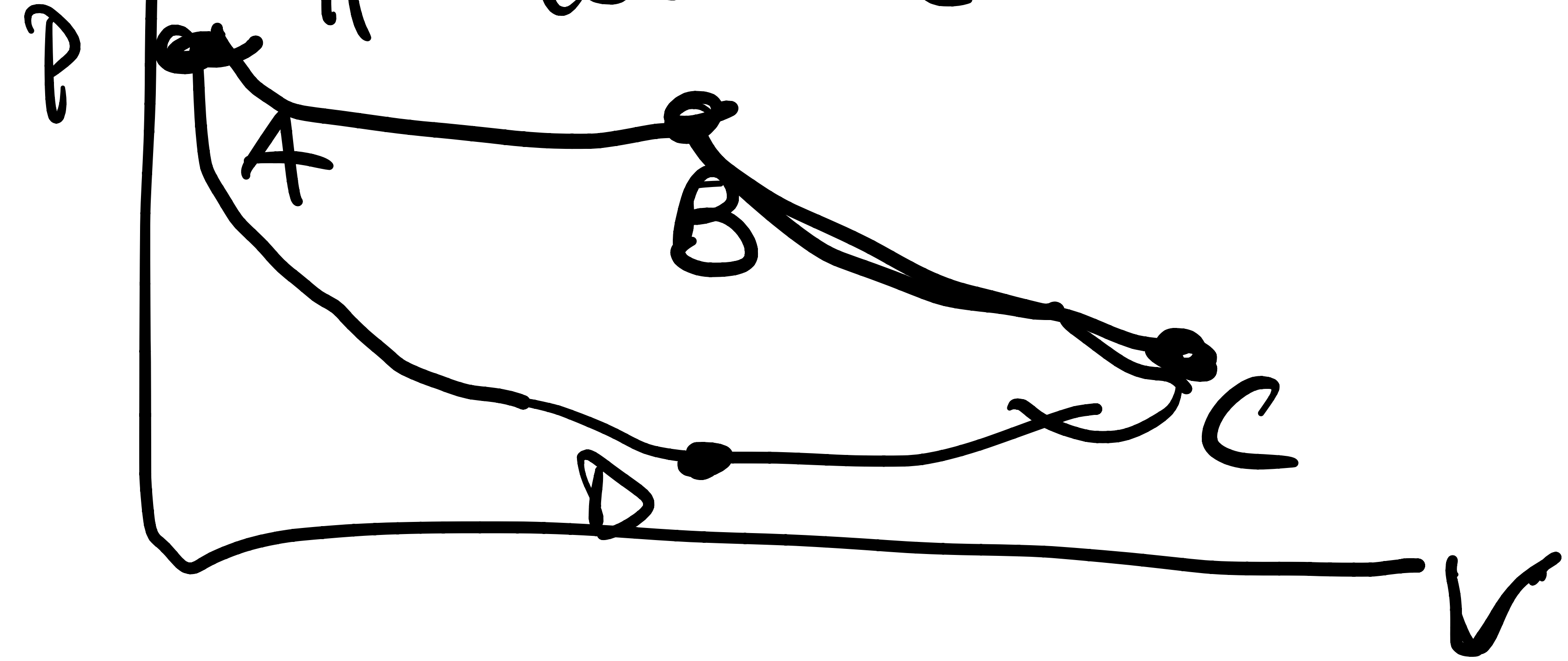


1. $dT = 0$ expansion @ T_H
2. $dq = 0$ expansion $T\downarrow$
3. $dT = 0$ compression $T_c = 0$
4. $q = 0$ compression

$$w^{sys} = -\oint PdV \rightarrow w_{done} = -w^{sys} = \oint PdV$$

$T_H = 200^\circ C \quad T_C = 20^\circ C$

$T_H = 400^\circ C \quad T_C = -10^\circ C$



Key heat in

	q_{sys}	w^{on}_{sys}	
(1) $dT=0$ @ $T=T_H$ $A \to B$	$nRT_{Hot}\ln\left(\frac{V_B}{V_A}\right)$	$-nRT_{Hot}\ln\left(\frac{V_B}{V_A}\right)$	$[dE=0]$
(2) $dq=0$ $B \to C$	0	$C_v(T_{cold} - T_{Hot})$	
(3) $dT=0$ $C \to D$ @ T_c	$nRT_{cold}\ln\left(\frac{V_D}{V_c}\right)$	$-nRT_{cold}\ln\left(\frac{V_D}{V_c}\right)$	
(4) $dq=0$ $D \to A$	0	$C_v(T_{Hot} - T_{cold})$	

$$w_{done} = -w_{sys} = nRT_{Hot} \ln\left(\frac{V_B}{V_A}\right)^{>0}$$

$$+ \underbrace{nRT_{Cold} \ln\left(\frac{V_D}{V_C}\right)}_{<0}$$

$$q_{in} = nRT_{Hot} \ln\left(\frac{V_B}{V_A}\right)$$

$$\epsilon = \frac{w_{done}}{q_{in}} = \frac{nRT_{Hot} \ln\left(\frac{V_B}{V_A}\right) + nRT_C \ln\left(\frac{V_D}{V_C}\right)}{nRT_{Hot} \ln\left(\frac{V_B}{V_A}\right)}$$

exercise → plug in adiabat formula

$$= 1 + \frac{T_{Cold} \ln\left(\frac{V_D}{V_C}\right)}{T_{Hot} \ln\left(\frac{V_B}{V_A}\right)} = -T_C/T_H$$

$$\epsilon = 1 - T_{cold}/T_{Hot} = \frac{T_{Hot} - T_{cold}}{T_{Hot}}$$

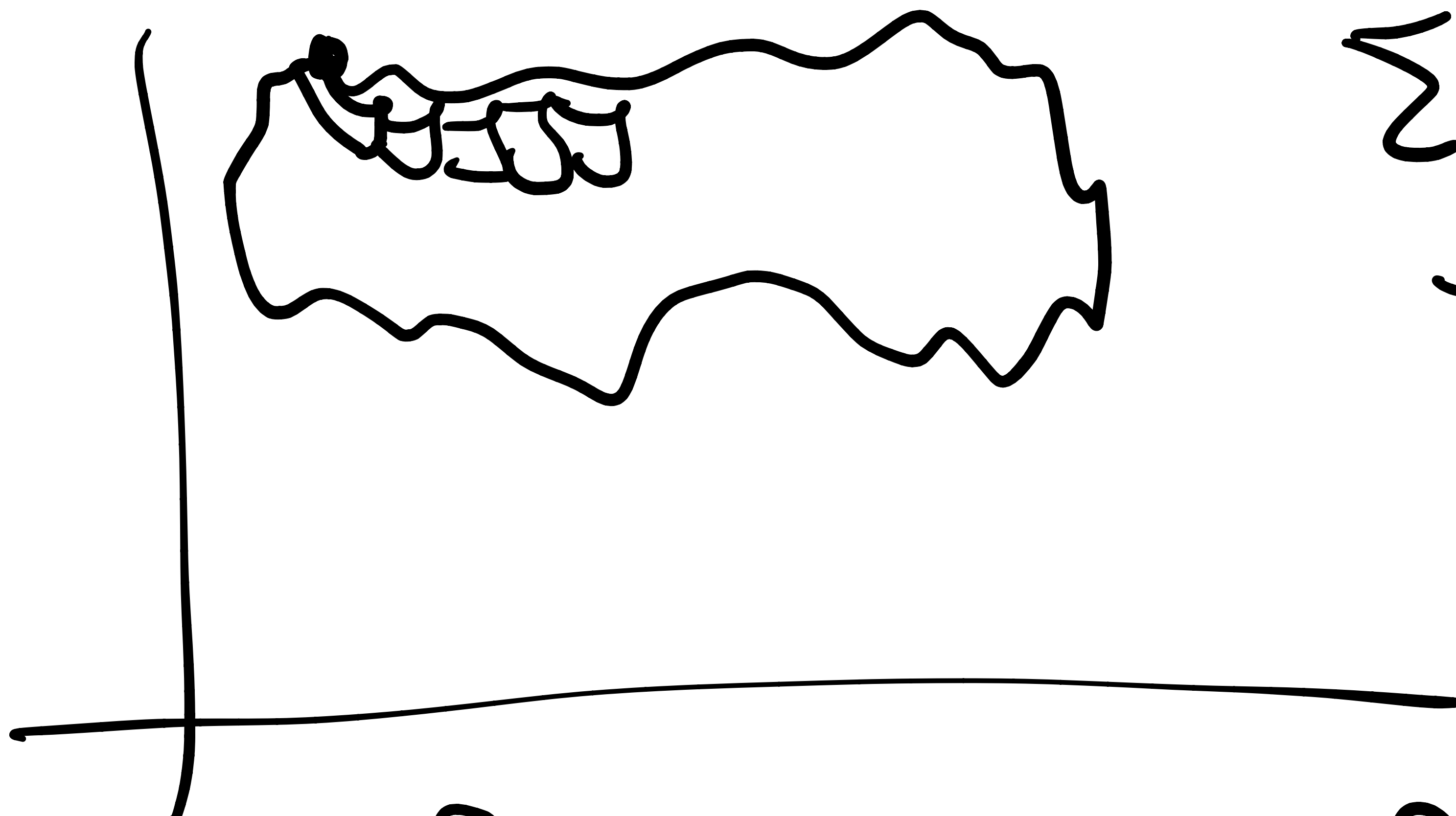
$$\frac{q_3}{q_1} = -\frac{T_C}{T_H} \Rightarrow \frac{q_1}{T_1} + \frac{q_3}{T_3} = 0$$

$$\oint \frac{dq}{T} = 0$$

define $dS = dq^{rev}/T$

entropy (pg 143)

S is a state function



$\sum$ carnot cycles to give any other cycle

$\Rightarrow \oint dS = 0$ for general cycles

What is ΔS for expansions of ideal gas

1. $dP = 0$ $\quad \Delta S = \int \frac{dq^{rev}}{T} = C_p \int \frac{dT}{T} = C_p \ln\left(\frac{T_f}{T_i}\right)$

$dq = C\,dT$

2. $dV = 0$ $\quad \Delta S = C_V \ln\left(T_f/T_i\right)$

plug in $PV = nRT$

3. $dT = 0,$ $\quad dq = -dw = P\,dV$

$$\Delta S = \int \frac{P\,dV}{T} \underset{\text{ideal gas}}{=} nR \int \frac{1}{V} dV = nR \ln\left(V_f/V_i\right)$$

(4) $dq = 0$ so $dS = 0$

$\Delta S = 0$

carnot cycle