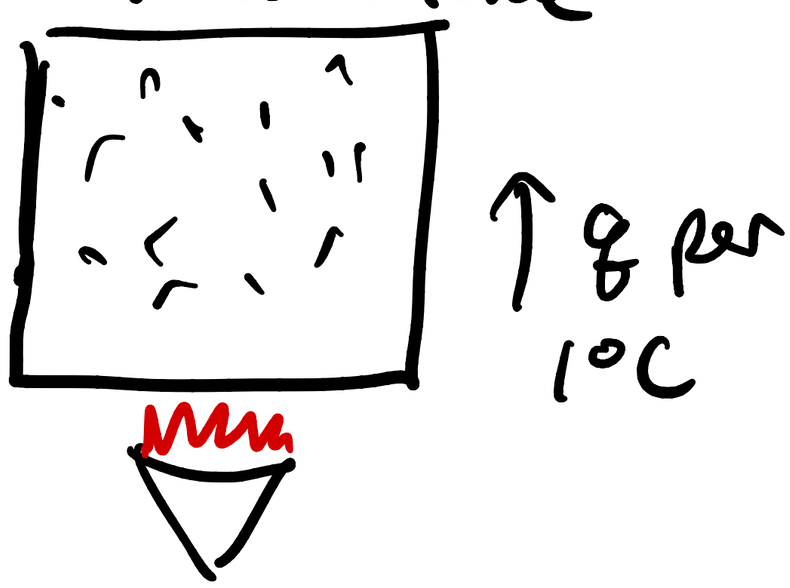
Lecture 6

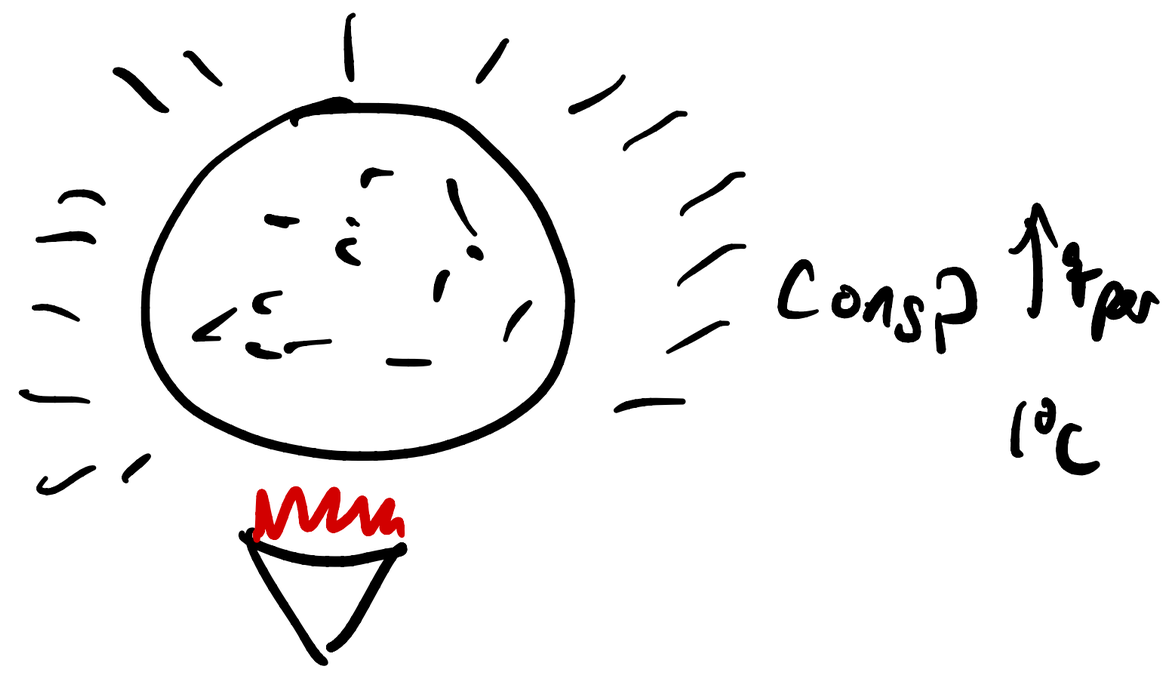
heat capacity (extensive quantity)

$$dq = C\,dT$$

$$C_V = \left(\partial q / \partial T\right)_V \qquad C_P = \left(\partial q / \partial T\right)_P$$

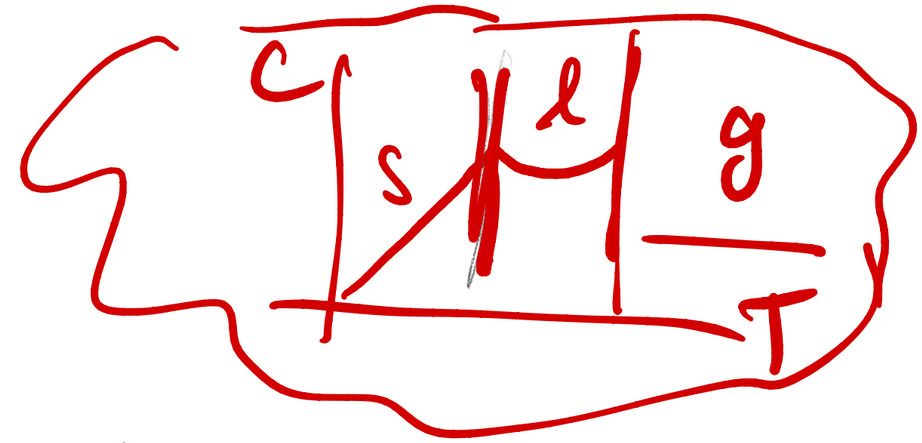
fixed volume





$$C_P > C_V$$

How much heat flows:



$$dq = C(T)\,dT$$

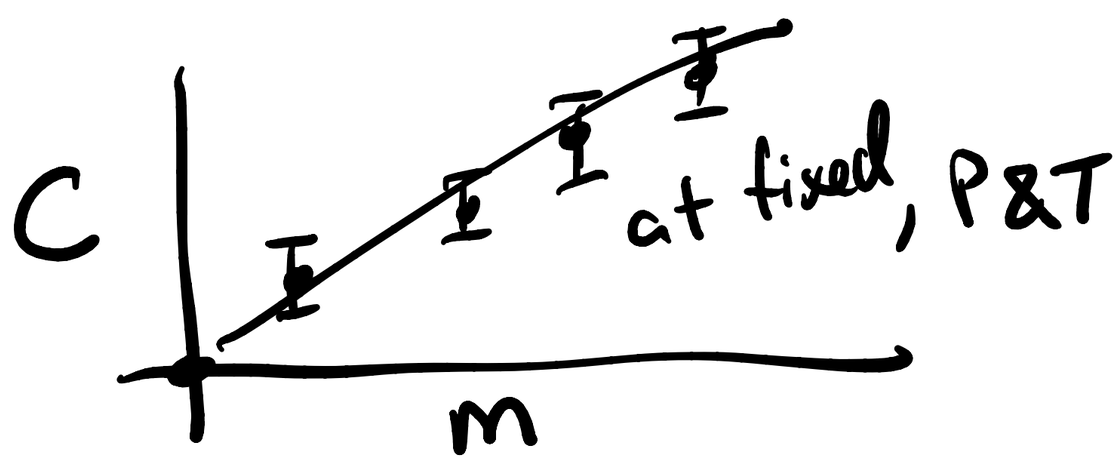
$$q = \int_{T_1}^{T_2} C(T)\,dT \approx C\int_{T_1}^{T_2} dT = C\,(T_2 - T_1)$$

if T_1 & T_2 aren't that diff

& phase transition, $C(T) \approx C$

"$q = nC\Delta T$" = "$q = mC\Delta T$"

C ↑ molar C ↑ per mass



How much does $\mathcal{E}$ change

$$d\mathcal{E} = dq + dw \qquad \text{(1st law)}$$

$$= C\,dT - P\,dV$$

$$\Delta\mathcal{E} = \int_a^b (C\,dT - P\,dV)$$

a) const volume, $dV = 0$ $\qquad C = C_V$

$$\Delta\mathcal{E} = C_V \Delta T$$

b) const P, $C = C_P$

$$\Delta\mathcal{E} = C_P \Delta T - P\Delta V$$

$$C_V = \left(\frac{\partial \varepsilon}{\partial T}\right)_V = C_V \frac{\partial T}{\partial T} = C_V$$

$$d\varepsilon = C_V dT$$

$$dq = d\varepsilon$$

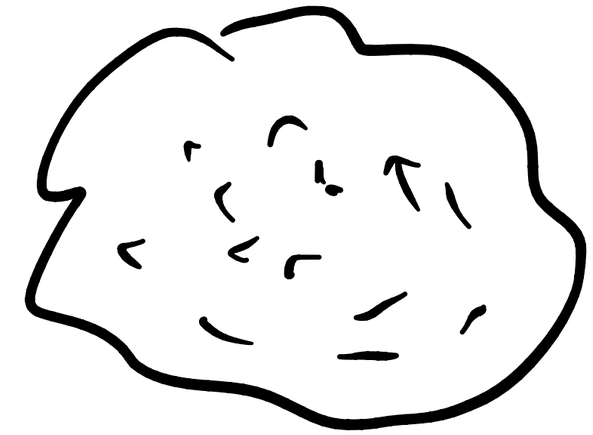
(a) C_V is how much energy changes for some change in temp

b) $d\varepsilon = dq - PdV \Rightarrow dq = d\varepsilon + PdV$

$$C_P = \left(\frac{d(\varepsilon + PV)}{dT}\right)_P \qquad d\varepsilon + PdV$$

Enthalpy

$$H \equiv E + PV$$



$$d(H) = d(\mathcal{E}) + d(PV)$$

$$= d\mathcal{E} + VdP + PdV$$

Consider $dP = 0$

@ const P

$$dH = d\mathcal{E} + PdV = dq$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_P$$

Ideal gas: (monatomic ideal gas)

2 assumptions: - take up no space

- don't interact

① $PV = nRT$

$$E_{total} = KE + \cancel{PE}^{\,0}$$

② $E = \frac{3}{2} nRT$ $\quad \propto T$

$$C_V^{ig} = \left(\frac{\partial E}{\partial T}\right)_V = \frac{3}{2} nR \quad , \quad C_V = \frac{C_V}{n} = 3/2\, R$$

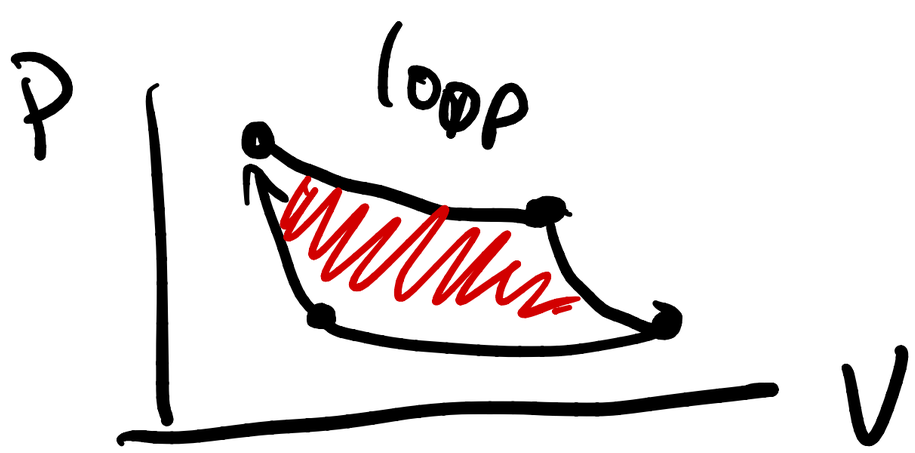
$$C_P^{ig} = \left(\frac{\partial (E+PV)}{\partial T}\right)_P = \frac{\partial}{\partial T}\left(\frac{3}{2} nRT + nRT\right) = \frac{5}{2} nR$$

$C_p > C_v$, ideal gas

$C_p - C_v = nR$

$\rightarrow$ true for any # atoms

Preview: how to do work through a cyclic process



$$W = \int_{cycle} -P\,dV$$

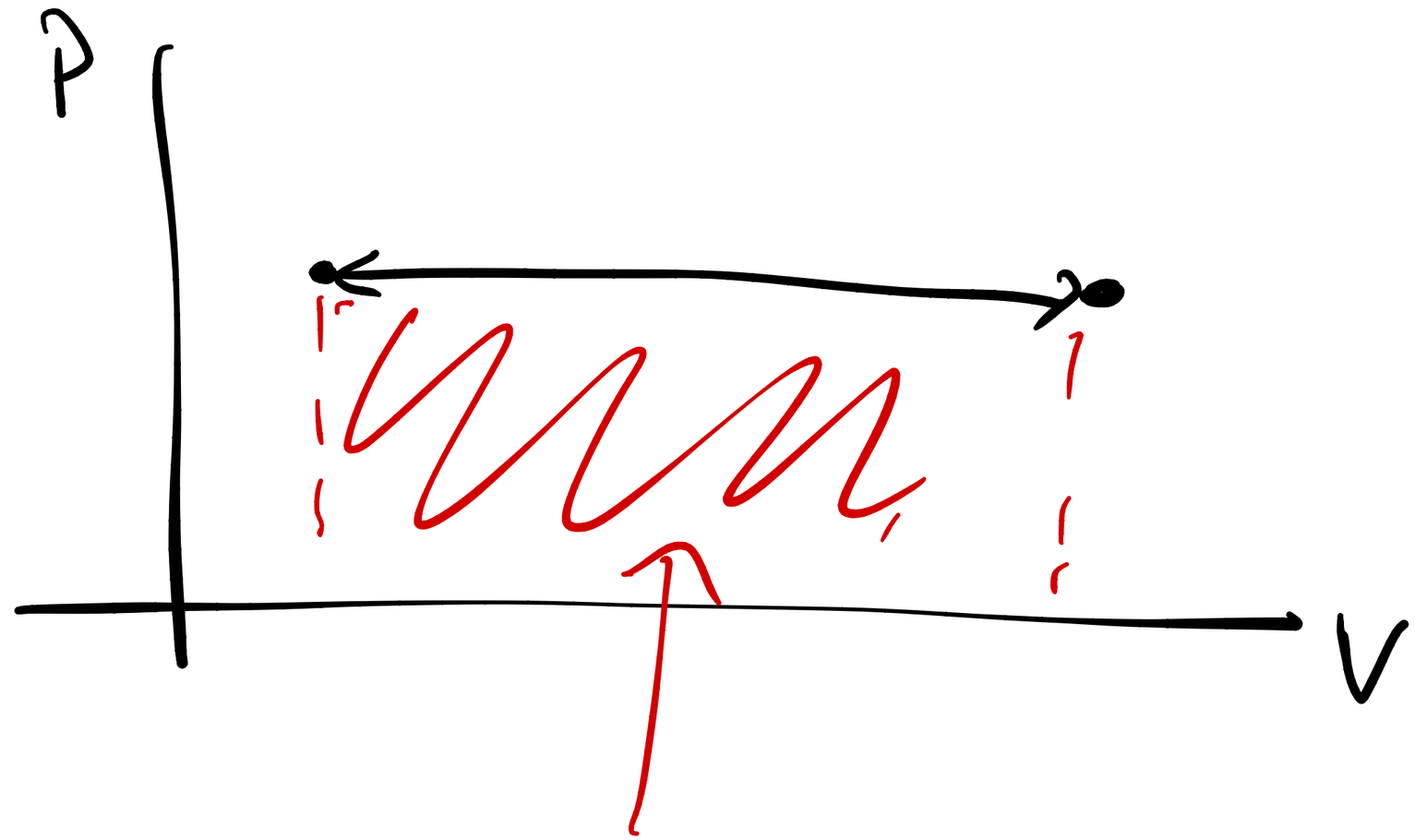
Change state:

- Const P, $dP=0$
- Cont V, $dV=0$
- Const T, $dT=0$
- Adiabatic, $dq=0$

Need w, q, ΔE

① dP

$(PV = nRT)$



true in general

$$\underline{w} = -P\Delta V$$

$$q = C_p \Delta T$$ ← for an ideal gas,

$$\Delta T = \frac{P_2V_2}{nR} - \frac{P_1V_1}{nR} = \frac{P}{nR}\Delta V$$

$$\Delta E = q + w = C_p\Delta T - P\Delta V$$

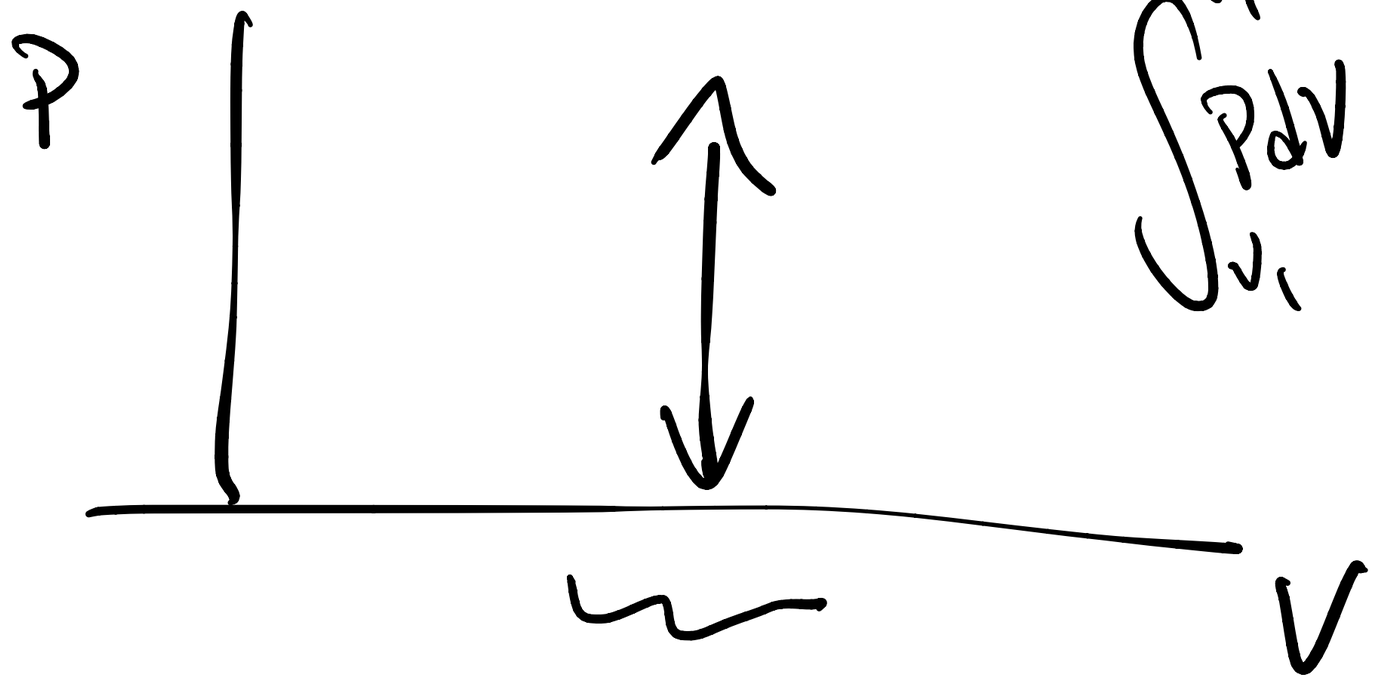
② $dV = 0$

$(dw = 0)$

$w = 0$

$q = C_V \Delta T$

$\Delta \mathcal{E} = C_V \Delta T$



$\int_{V_1}^{V_1} P\,dV$

no area under curve

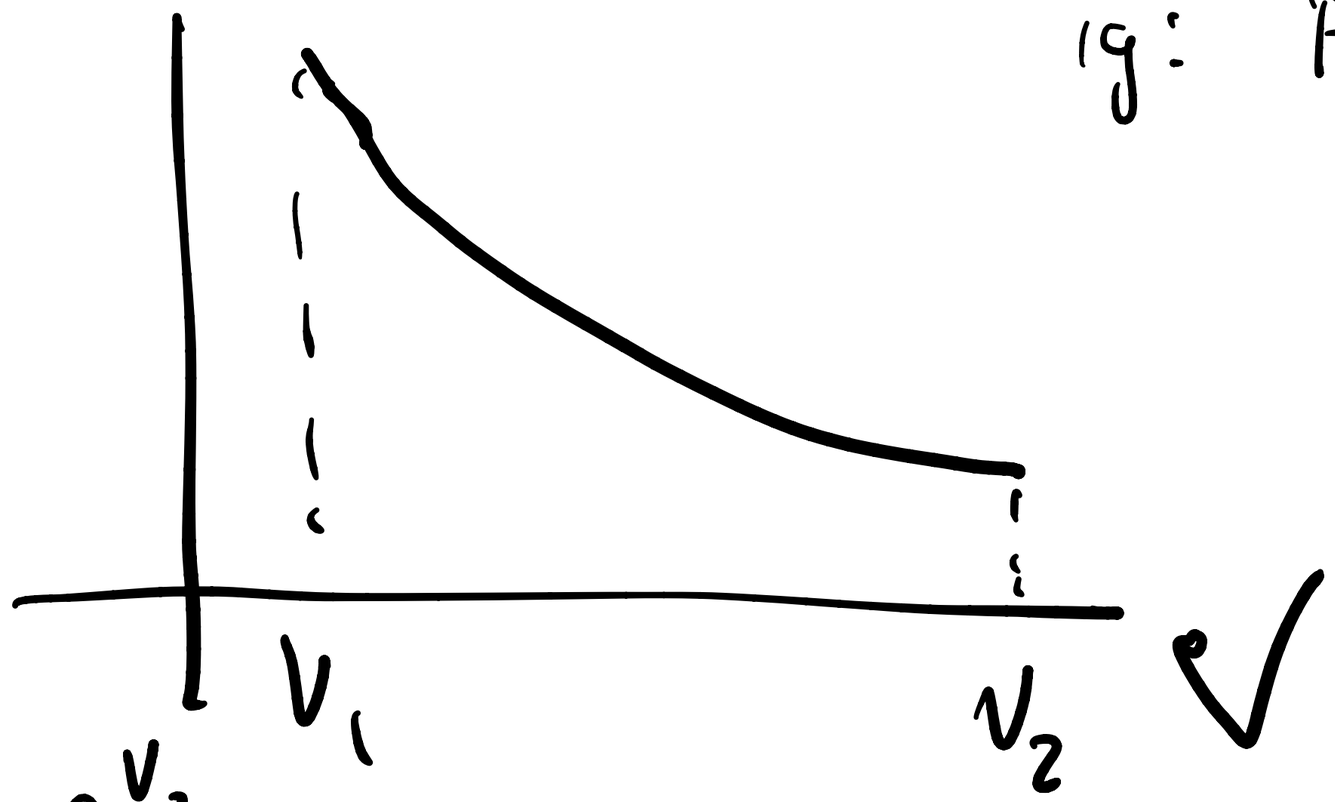
(for an ideal gas, follows from energy formula)

③ $dT = 0$

What does this look like

ig: $P = nRT \cdot \frac{1}{V}$

isothermal expansion or compression



$$w = -\int_{V_1}^{V_2} P\,dV = -\int_{V_1}^{V_2} \frac{nRT}{V}\,dV$$

$$= -nRT \int_{V_1}^{V_2} \frac{1}{V}\,dV$$

$$= -nRT \ln(V_2/V_1)$$

question

$q > 0$,

$q < 0$,

$q = 0$

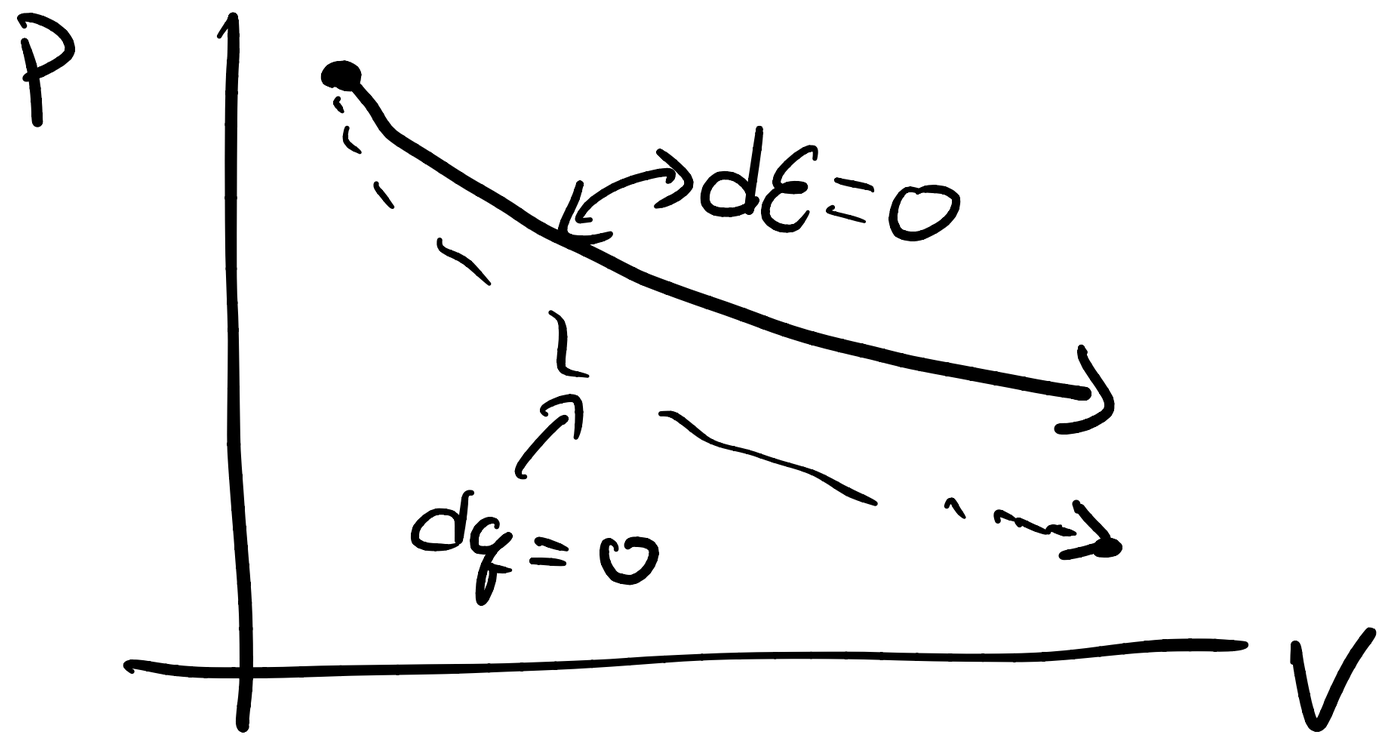
i.g. $$dE = d\left(\frac{3}{2}nRT\right) = \frac{3}{2}nR\,dT = 0$$

$$0 = dq + dw = q + w \Rightarrow q = -w$$

$$q = +nRT\ln\left(V_2/V_1\right)$$

$q > 0$ for expansion

④ Adiabatic expansion, $dq = 0$

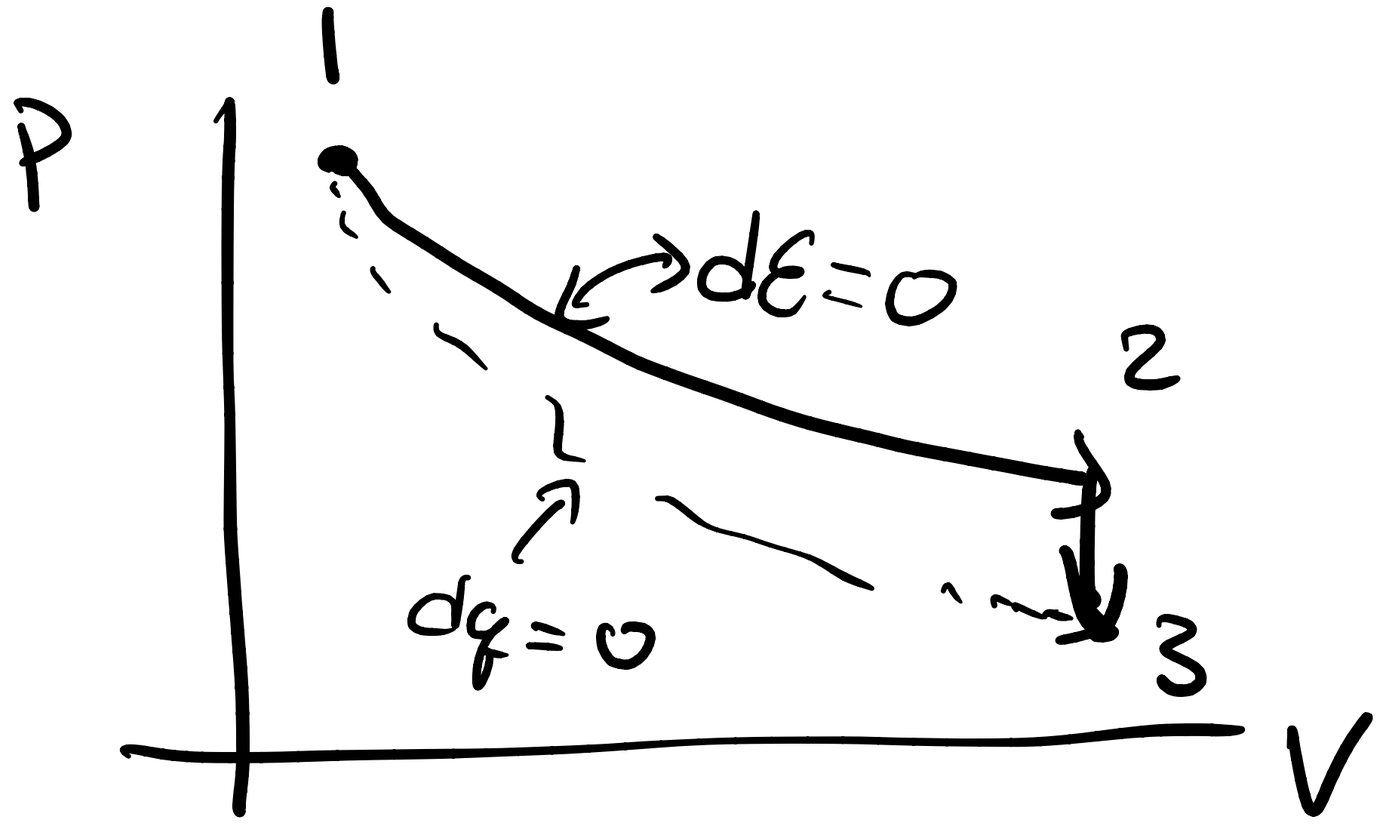


pressure drops faster due to cooling

$$d\mathcal{E} = \cancel{dq}^{\,0} + dw = -P(V)\,dV$$

$$= -\frac{nR}{V}T(V)\,dV$$

how much does it cool? ← next time



$1 \to 2 \quad dT = 0$

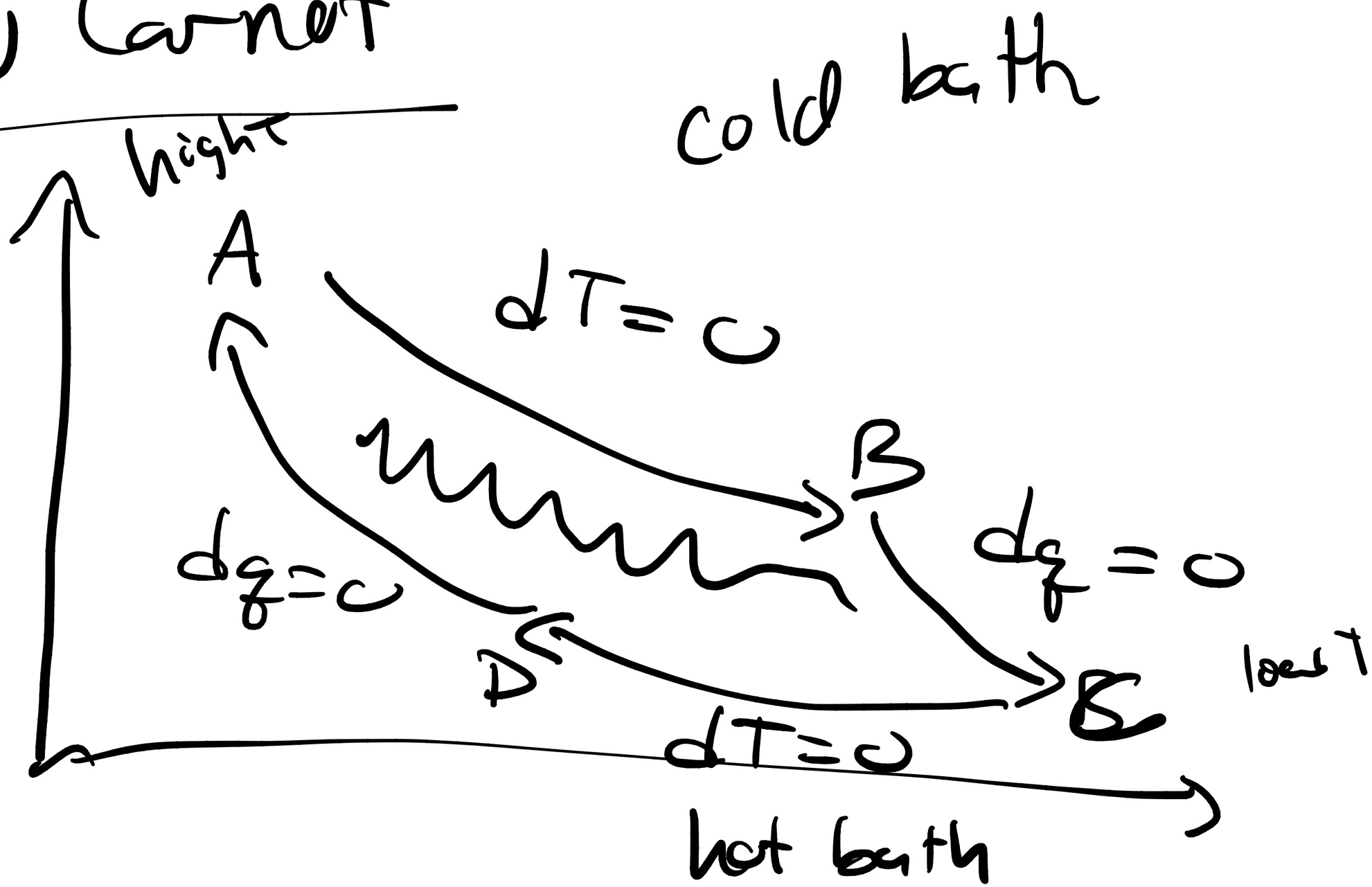
$2 \to 3 \quad dV = 0$

Change in E from Pt $1 \to$ Pt 3

$$= Pt_{1\to 2} + Pt_{2 \to 3}$$

next time ...

Preview Carnot



4 steps

temps T_h & T_c

efficiency $= w/q_{in} = 1 - T_c/T_h$