Lecture 3

Distributions – function that represents the chance of seeing a particular outcome out of all possible outcomes

$P(X) \leftarrow 0 \leq P(x) \leq 1$ for any x

$\sum_x P(X) = 1 \leftarrow$ "normalized"

Characterize a distribution:

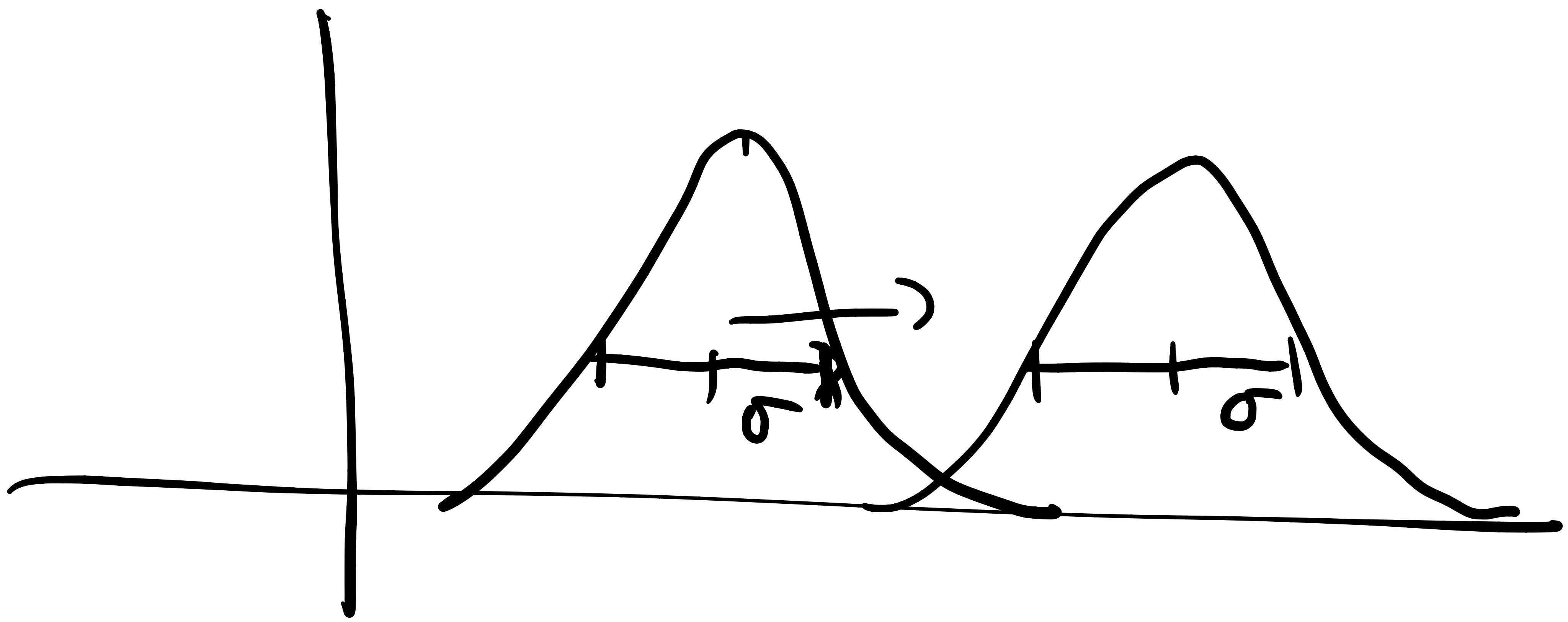
"moments" $\sum x^n P(X)$, n th moment

Eg $\mu = \sum x P(x)$ 2nd $\sum x^2 P(x)$

$\mu = \langle x \rangle = \sum x P(x)$

2nd moment $= \langle x^2 \rangle = \sum x^2 P(x)$

$\mathrm{Var}[x] = \sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$



Same σ, diff $\langle x^2 \rangle$

Binomial distribution

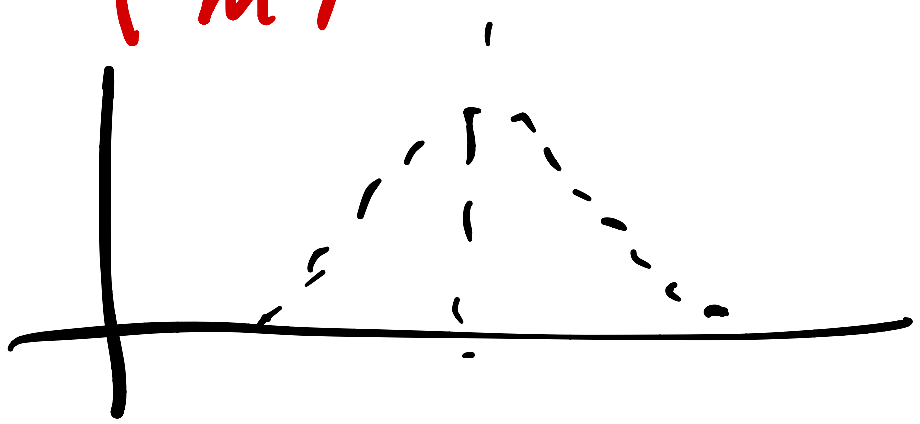
Chance of m events in N independent trials

$$P(m;N) = \binom{N}{m} p^m (1-p)^{N-m}$$

p is a chance on a single trial

$$\binom{N}{m} = \frac{N!}{(N-m)!\,(m)!}$$

Max $\binom{N}{m} \leftarrow m = \frac{N}{2}$



1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Max

Discrete distribution 2:

Poisson Distribution, limit of the Binomial distribution

Chance of n events in some fixed interval or area, in limit $p \to 0$ "rare events"

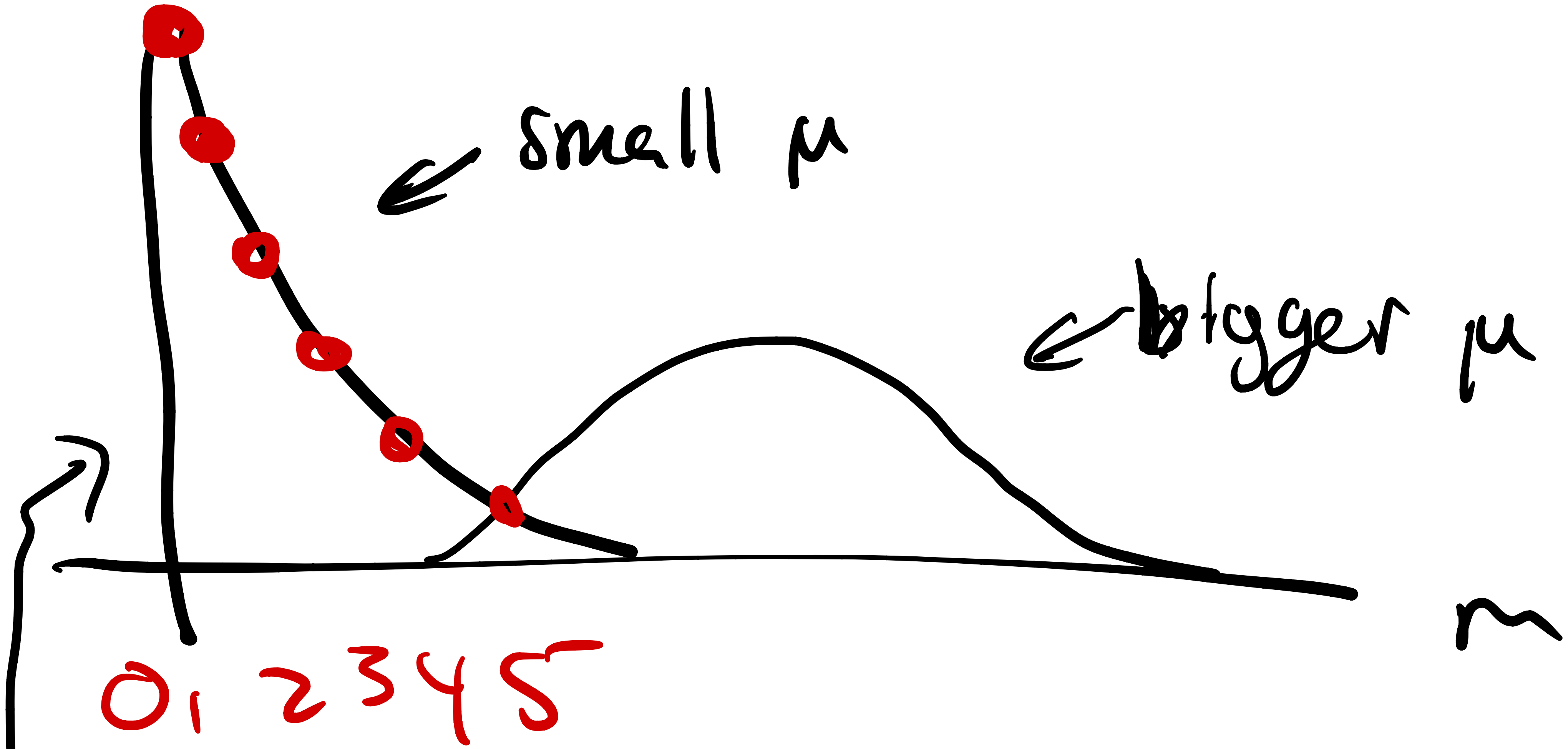
limit $N \to \infty$, $p \to 0$

$$P(n; \mu) = \frac{\mu^n e^{-\mu}}{n!}$$ ← μ is mean

unitless

Mean μ, Variance $= \mu$

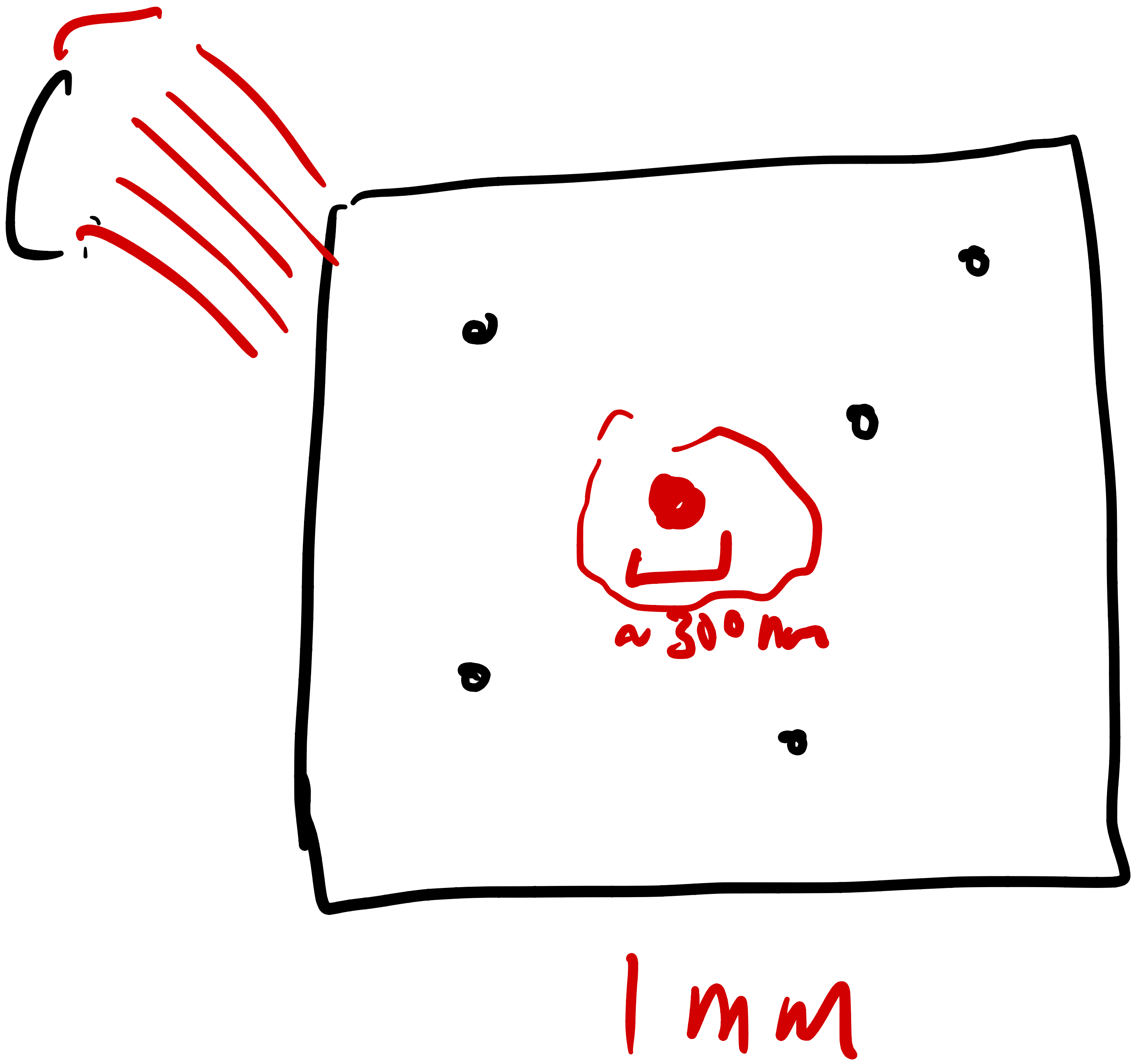
$$P(n) = \frac{e^{-\mu}\mu^n}{n!}$$

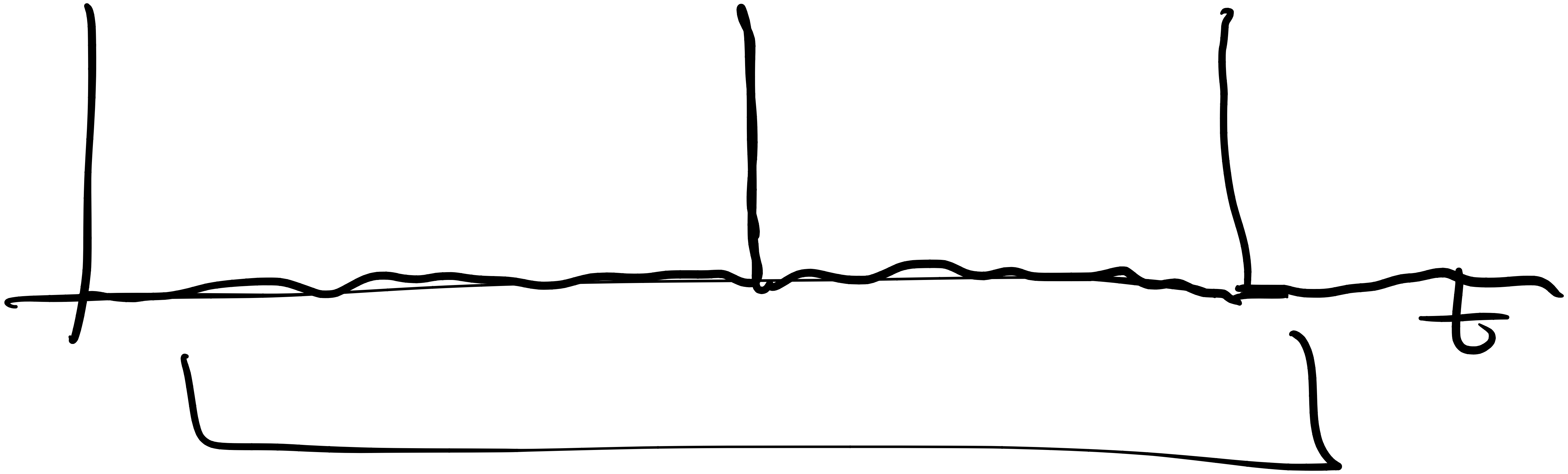


most likely # is 0

dilute solution

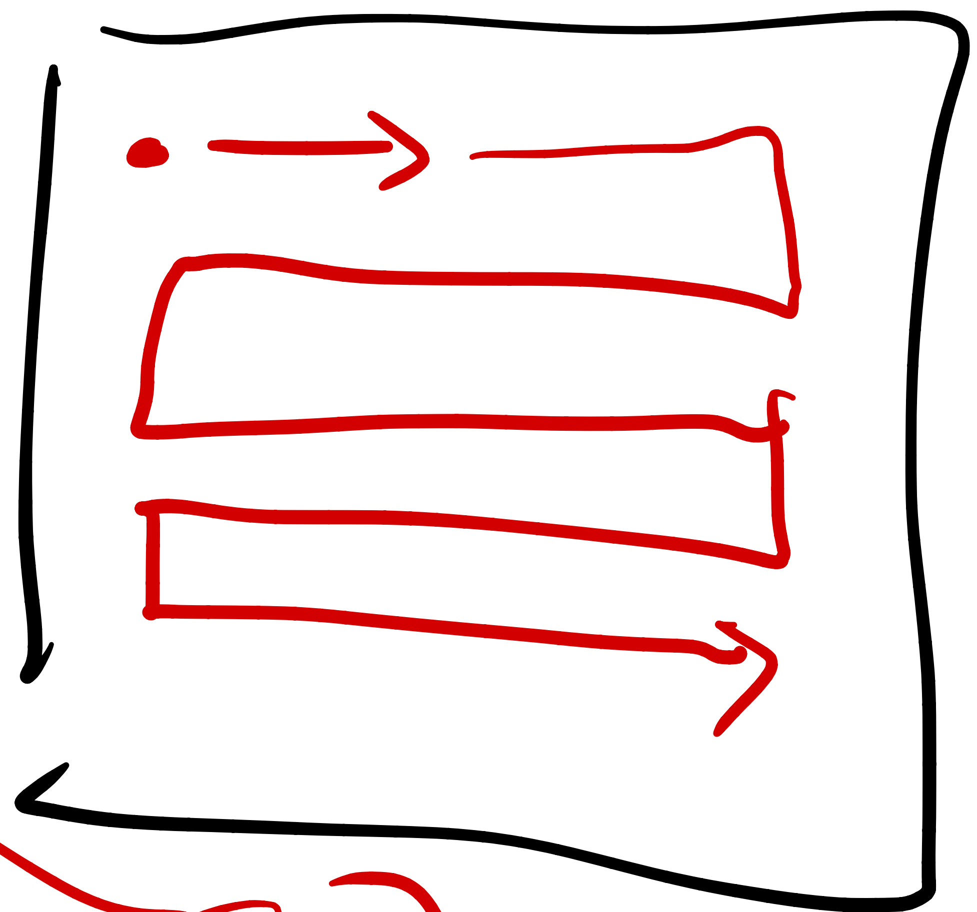
detect events





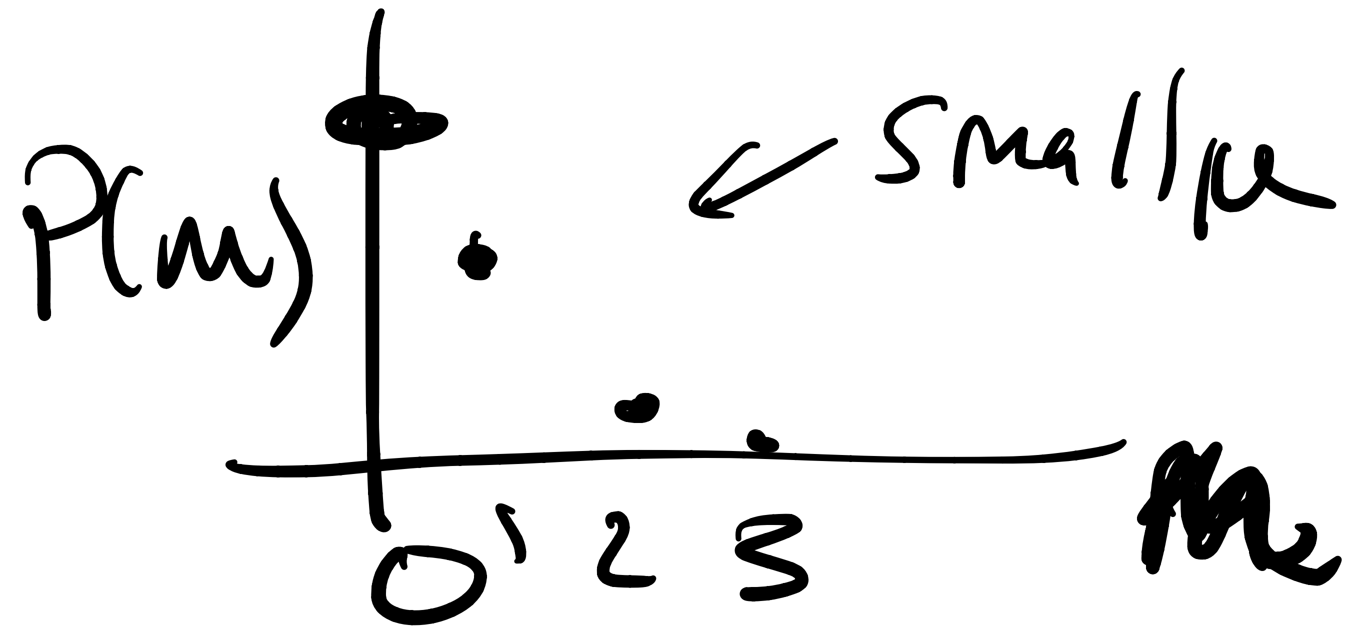
fixed interval, poisson dist

conc



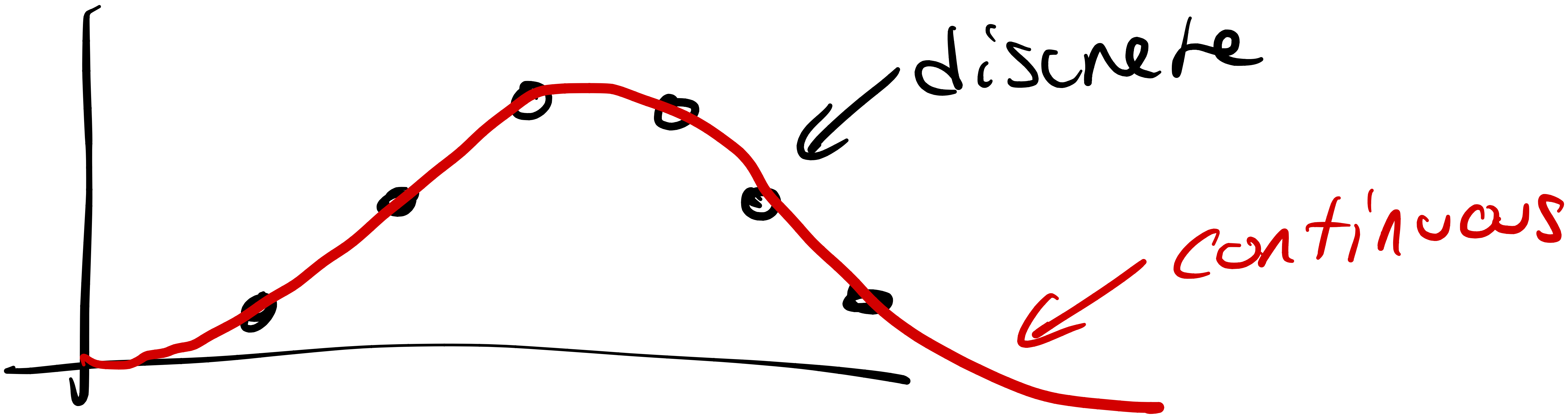
$$\text{Area} \cdot \frac{\#}{\text{Area}} = \mu$$

Chance that I see 2 molecules in 1 gridspot



Preview continuous distribution

$P(x)$ x can be any real #



① $P(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ Normal

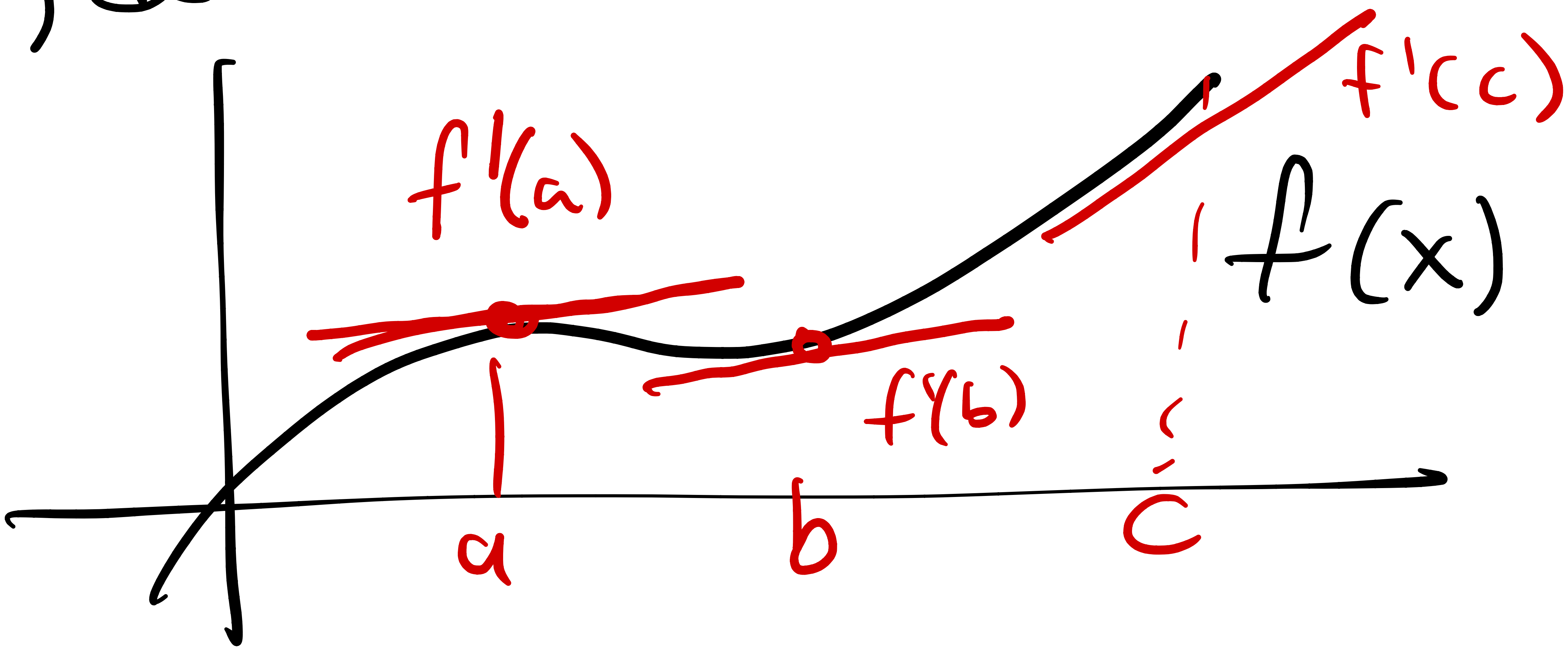
$N(x; \mu, \sigma)$

② $P(x) = \dfrac{1}{\lambda} e^{-\lambda x}$, $\mu = \dfrac{1}{\lambda}$

Calculus review

(1) derivative, function gives you slope of a function at a point

i) derivative is the slope

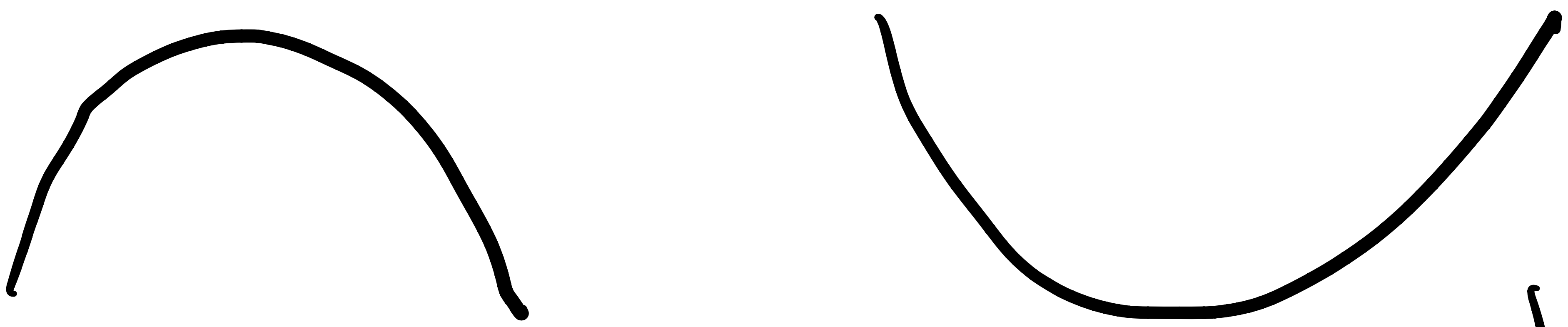


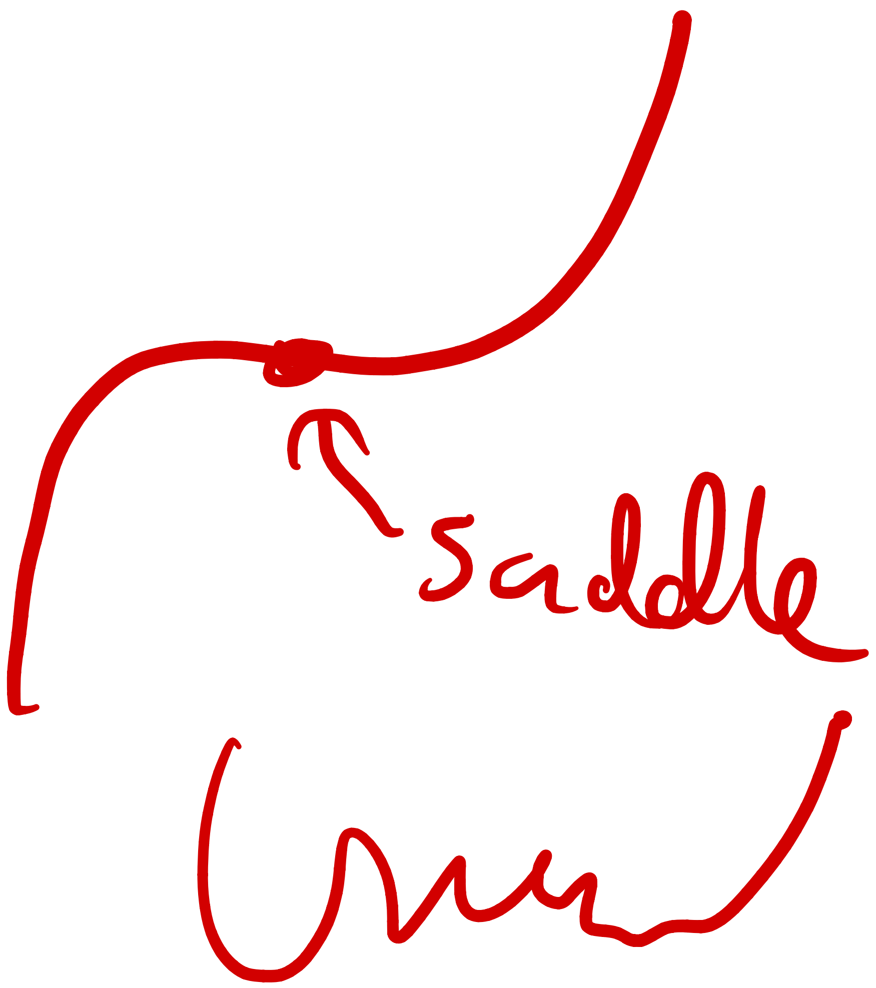
$$f'(x) = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

ii) max, min must have

$$\left.\frac{df}{dx}\right|_a = 0 \quad \text{if max or min}$$

b/c slope has to change sign





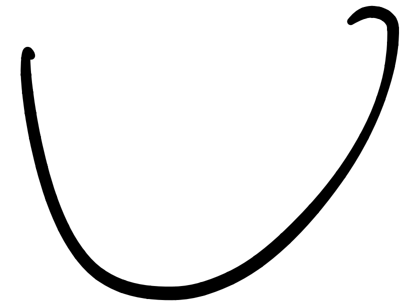
saddle $\frac{df}{dx} = 0$

max, min

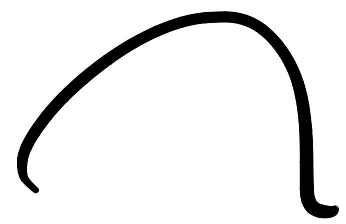
$$\frac{d}{dx}\frac{df}{dx} = \frac{d^2 f}{dx^2}$$

think about

$$f(x) = x^2 \xrightarrow{\frac{df}{dx}} 2x \xrightarrow{\frac{d^2f}{dx^2}} = 2$$



$$f(x) = -x^2 \rightarrow -2x \rightarrow -2$$



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$$g(x) = x^n$$

$$\frac{dg}{dx} = n x^{n-1}$$

$$g(x) = e^{ax}$$

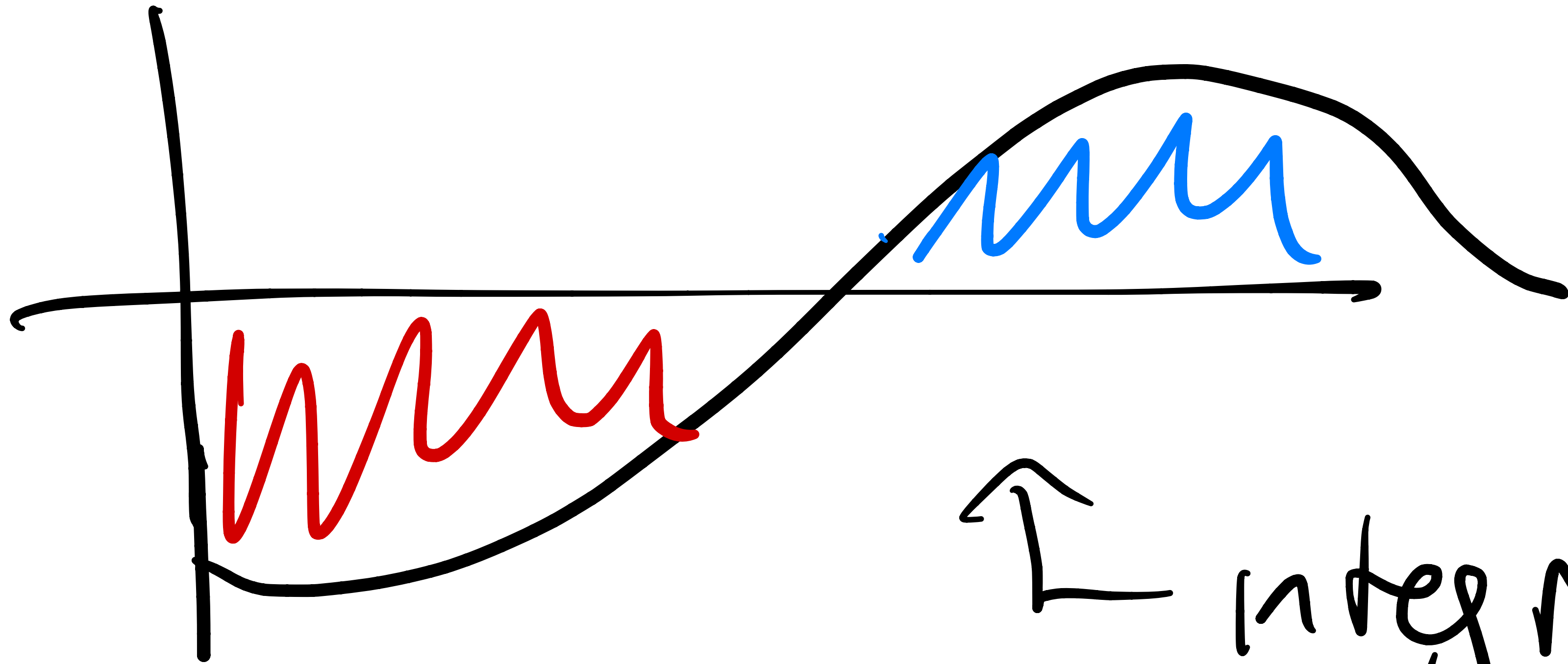
$$\frac{dg}{dx} = a e^{ax}$$

$$g(x) = \ln(x)$$

$$dg/dx = \frac{1}{x}$$

deriv of constant is 0

Integrals "Area under a curve"



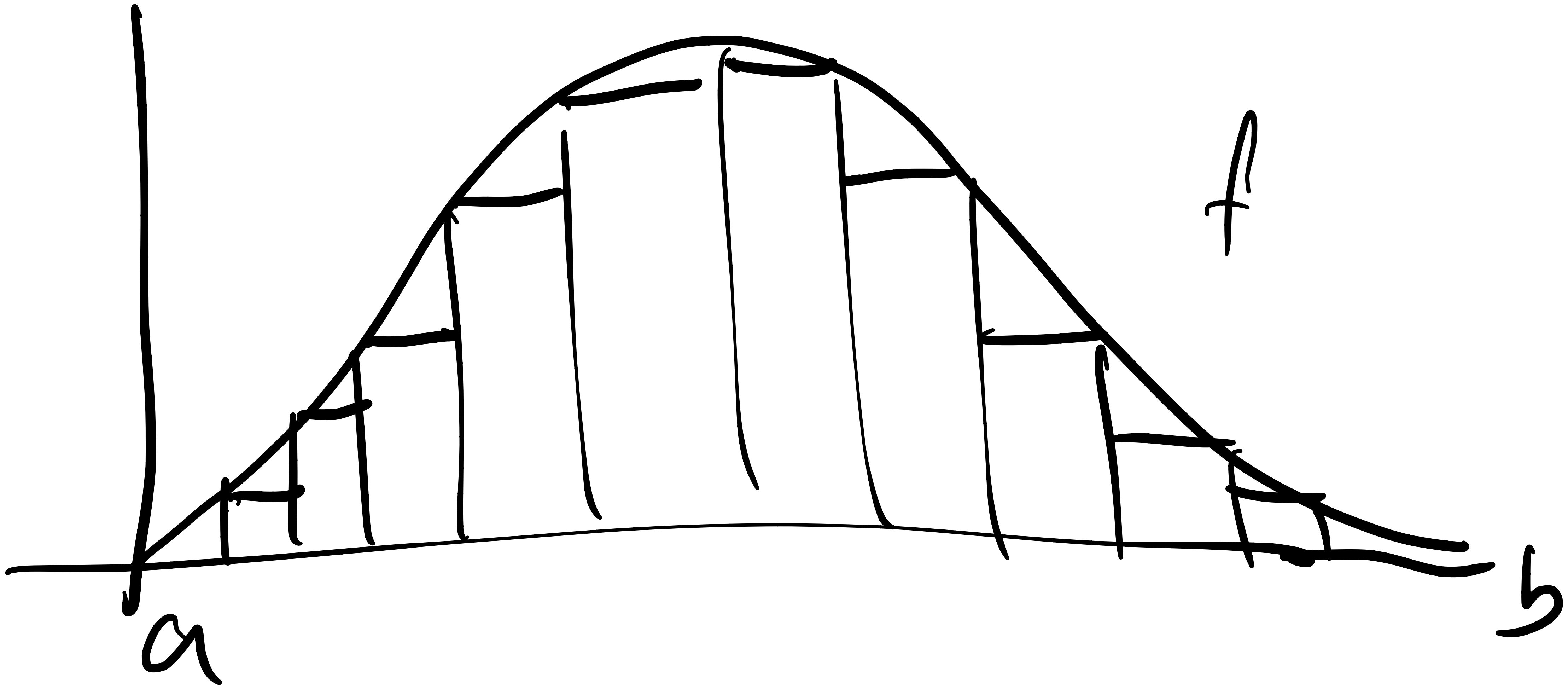
integral = $\boxed{} - \boxed{}$

anti derivative

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

derivative - lose information

$$\int \frac{df}{dx}\,dx = f(x) + C$$



① max Δx small

② Δx has units

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} f(x_i)\,\Delta x_i$$

$$= \frac{b-a}{N} \sum_{i=1}^{N} f(x_i)$$

cts distributions:

$$\langle x^n \rangle = \int_{-\infty}^{\infty} x^n P(x)\, dx$$

"domain" of $P(x)$?

P zero outside some region

$$\int_a^b x^n P(x)\, dx$$

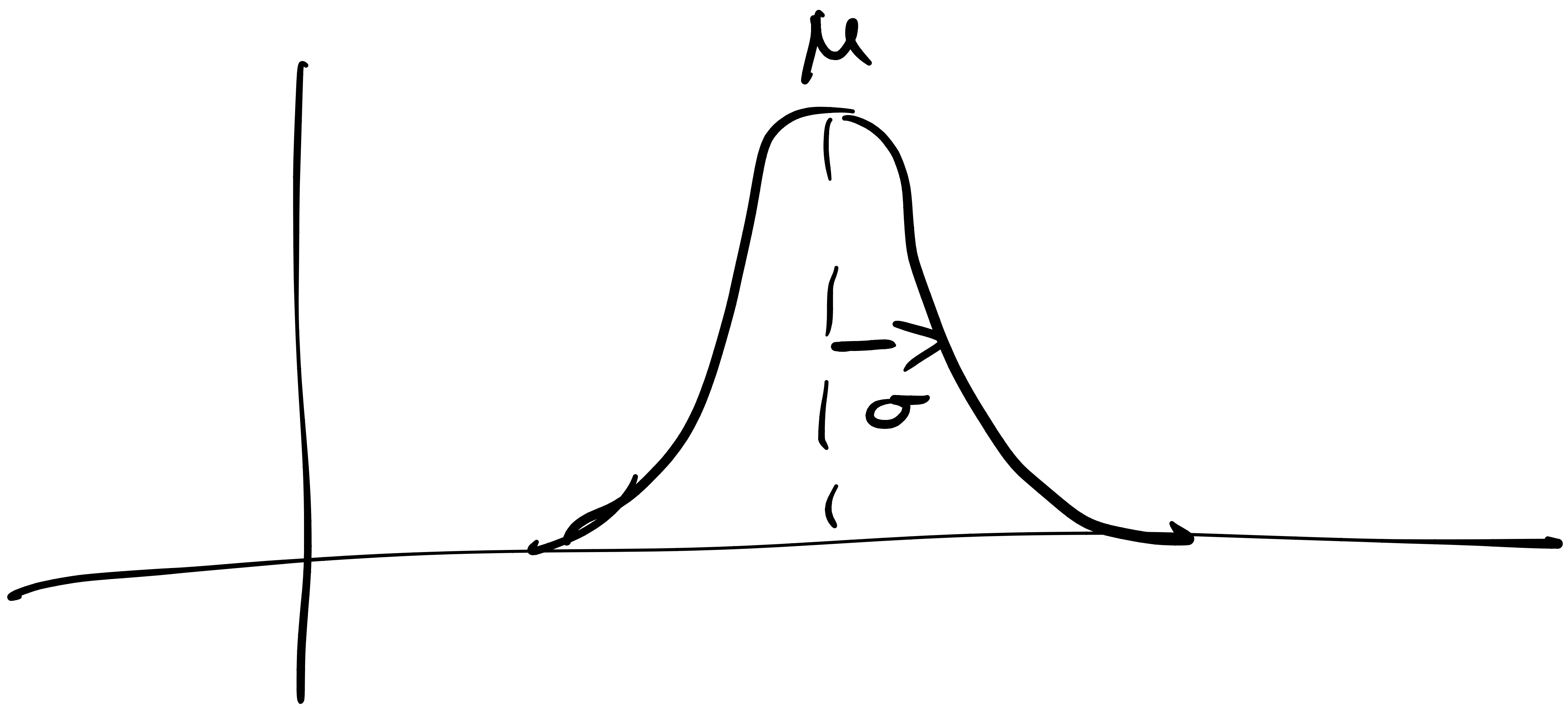
if $P(x) = 0$ for

$x > b$

$x < a$

$$Var(x) = \int_{-\infty}^{\infty} (x-\mu)^2 \, P(x) \, dx$$

$$Prob\,(a \le x \le b) = \int_a^b P(x) \, dx$$



Preview: product rule

$$\frac{d}{dx}(fg) = \frac{df}{dx}(x)\, g(x) + f(x)\frac{dg}{dx}(x)$$

differentials

"multiply dx"

$$d(fg) = (df)\, g + f\,(dg)$$