

# Lecture 11 - Phase Transitions

Eg of combined quantities in mixtures:

$$V = \sum_{i=1}^k \bar{V}_i n_i$$

if ideal,  $\bar{V}_i = \bar{V}_i^* = 1/\rho$

$$\rho = \frac{n}{V}$$

$$E_{\text{tot}} = \sum \bar{E}_i n_i$$

$$\left[ \begin{array}{l} P = \rho RT \\ \text{(ideal gas)} \end{array} \right]$$

$$\star G_{\text{total}} = \sum \bar{G}_i n_i = \sum M_i n_i$$

Aside

Partial pressure

$$P_i = X_i P_{\text{total}} \quad (\text{definition})$$

$$X_i = \frac{n_i}{\sum n_i}$$

for ideal gasses  $P_i \approx$  ideal gas pressure

$$\begin{aligned} (\text{Dalton's}) \quad P_{\text{total}} &= \frac{RT}{V} \sum_i n_i \\ &= \frac{n_i}{V} RT \end{aligned}$$

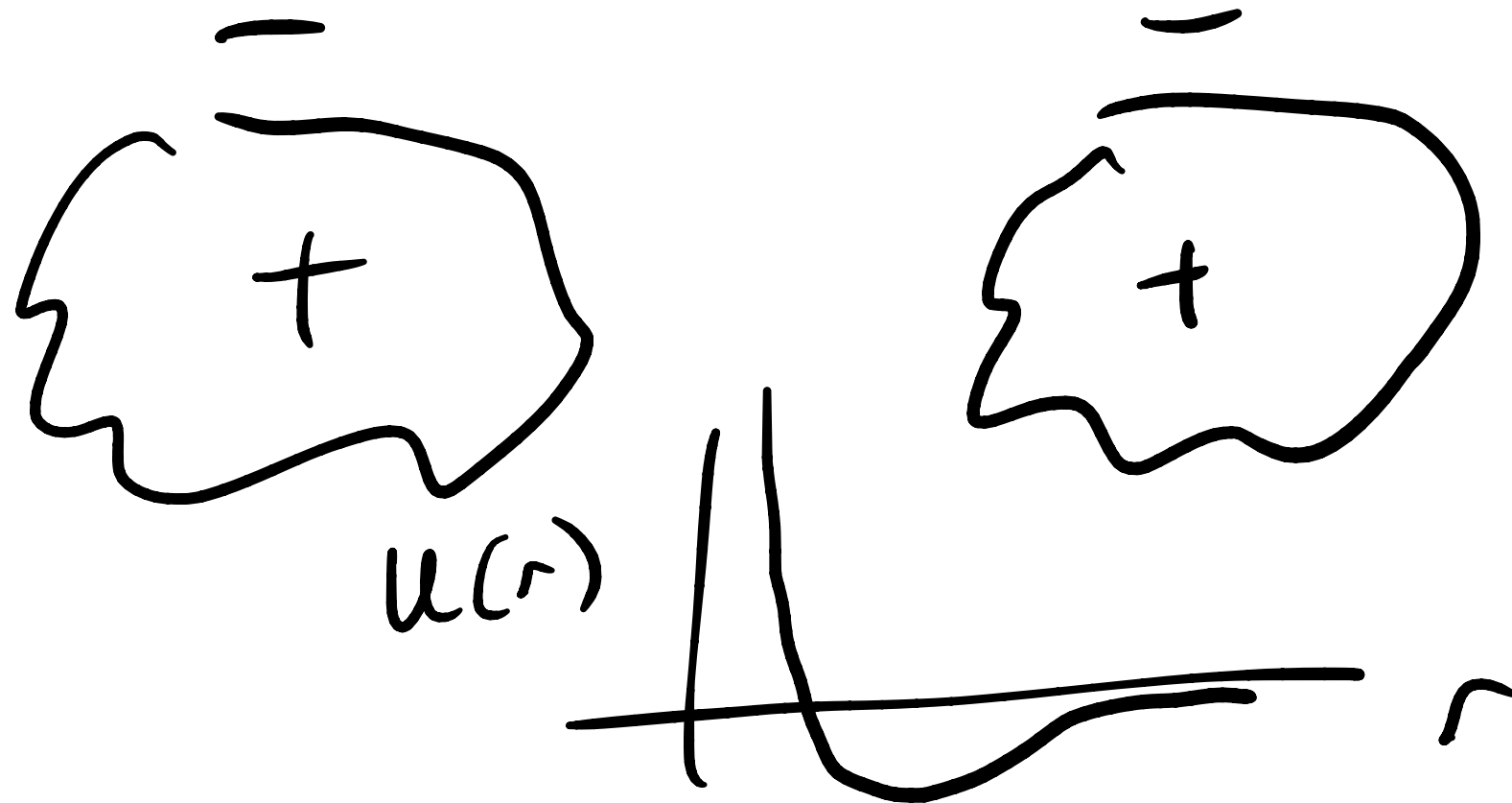
# Non ideal gasses: (cf Van-der Waal's

eqn of state)

2 contributions to interactions:

- "volume exclusion"  $\leftarrow$  pressure  $\uparrow$  (relative to an ideal gas)
- attractions  $\rightarrow$  pressure  $\downarrow$  (relatively)

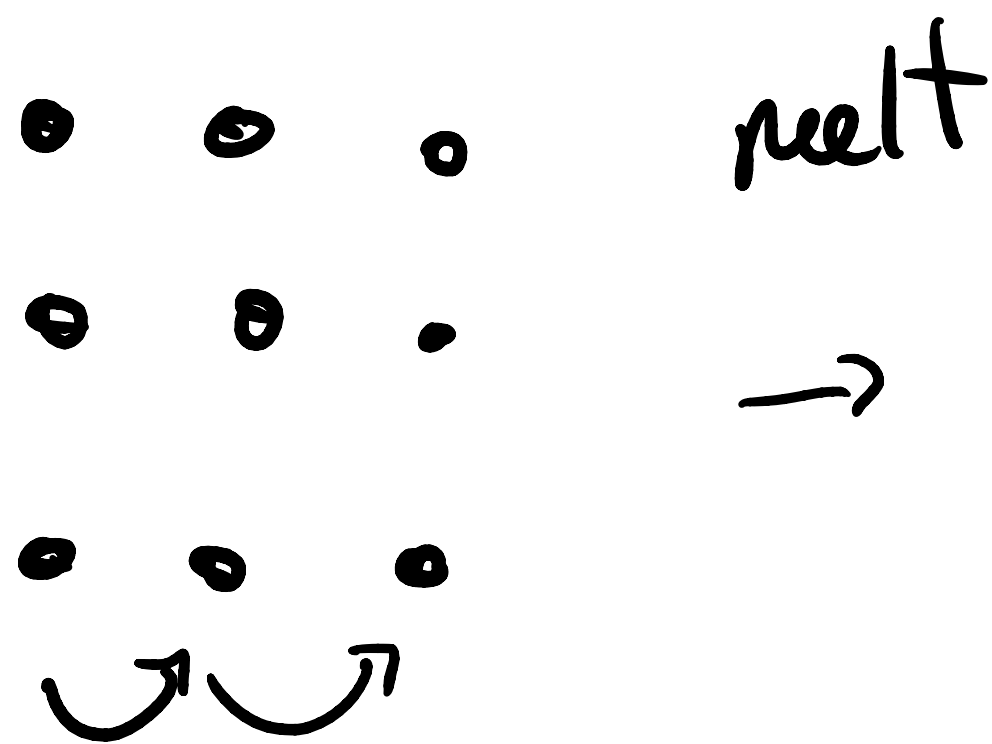
VDW:



# Phase transitions

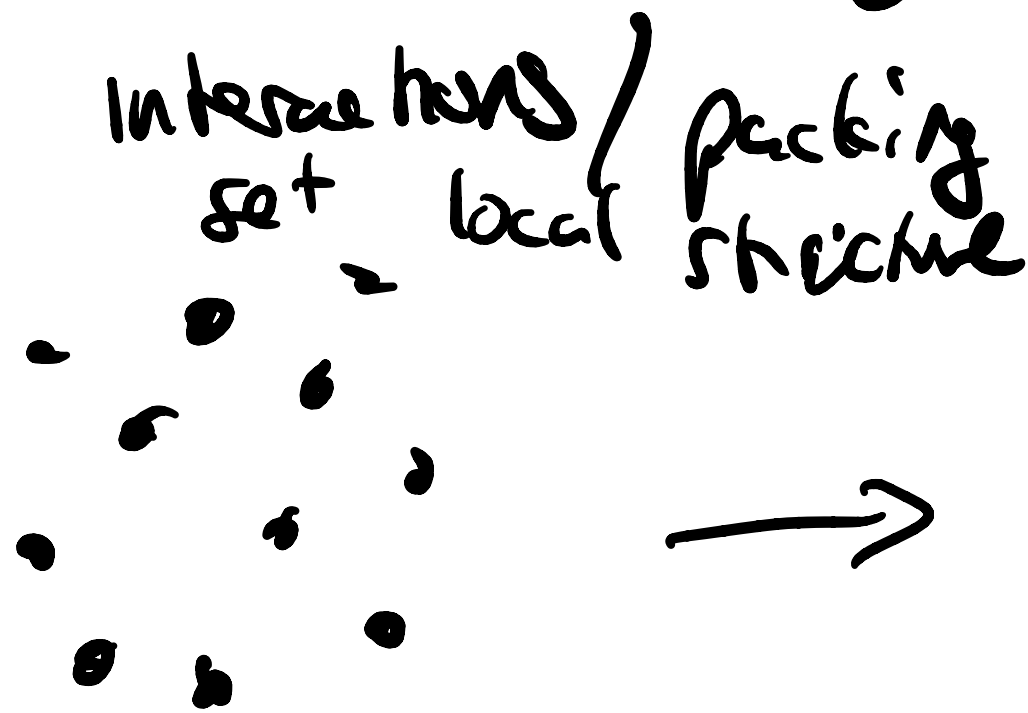
"Phases of matter": solids, liquids, gasses

Difference: is about symmetry (order)



long range order  
"very" ordered

melt  
→



less order  
local order

→

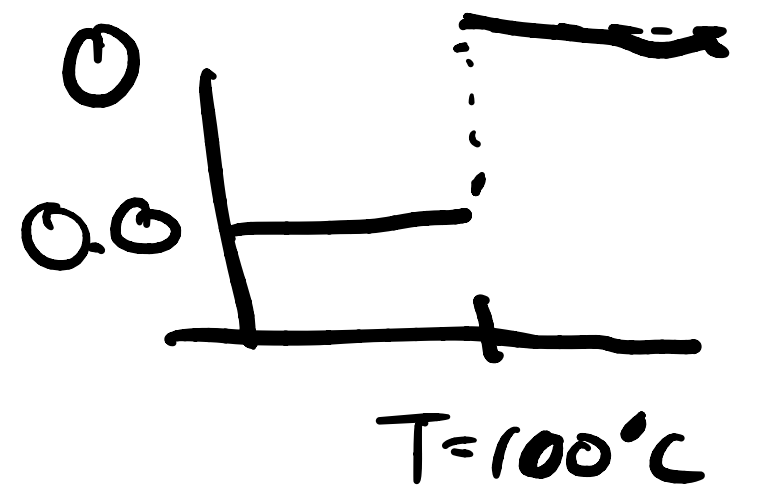


gas

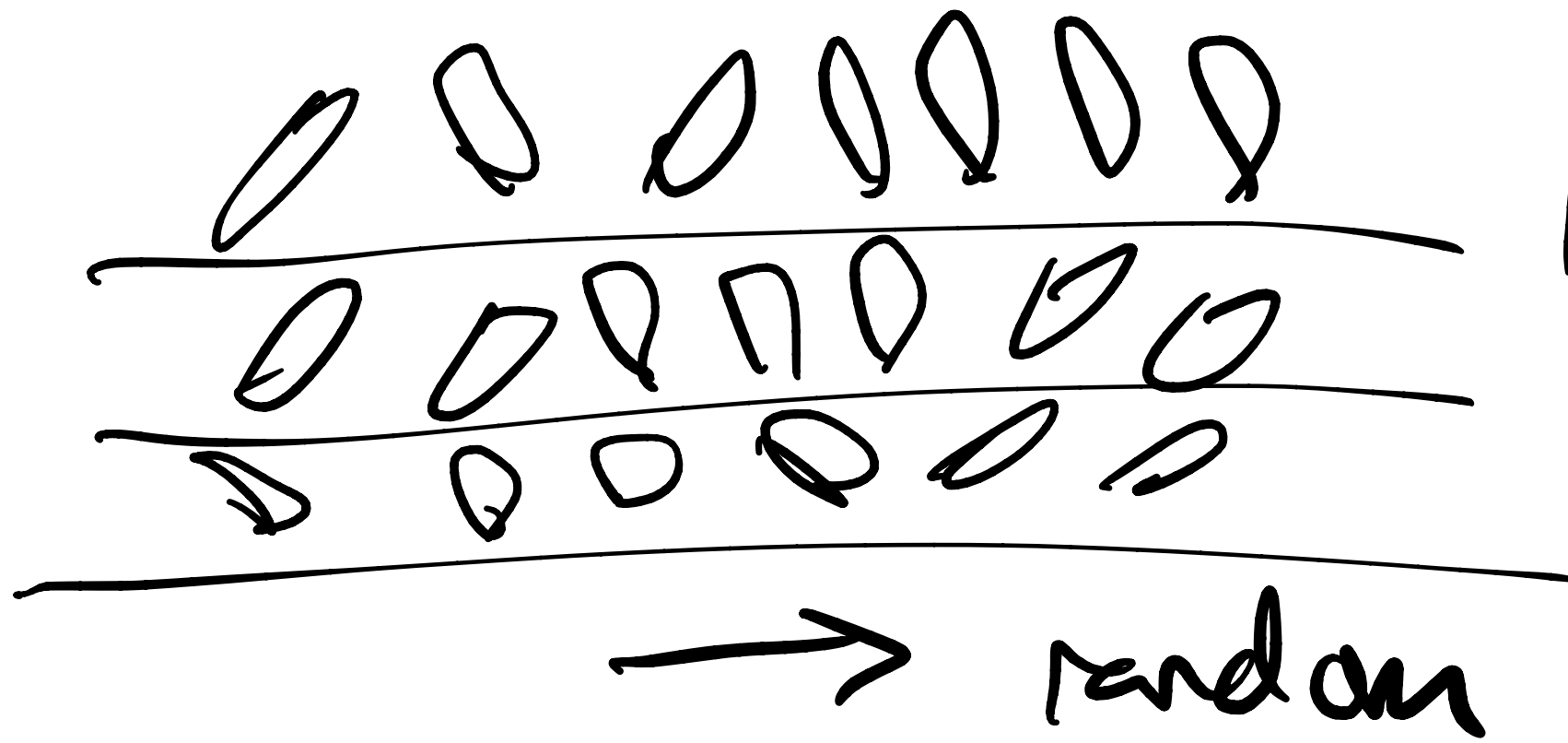
Define an "order parameter" that distinguishes different phases

Eg liq / gas  $\alpha(\rho) = \frac{\rho - \rho_L}{\rho_L}$

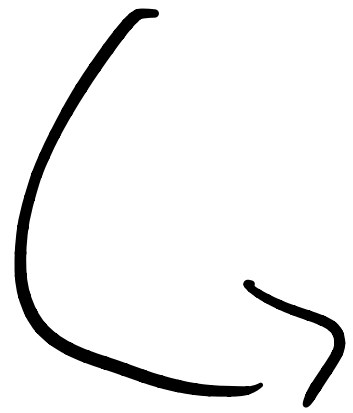
$$\alpha(\bar{v}) = \frac{\bar{v} - \bar{v}_L}{\bar{v}_L}$$



eg



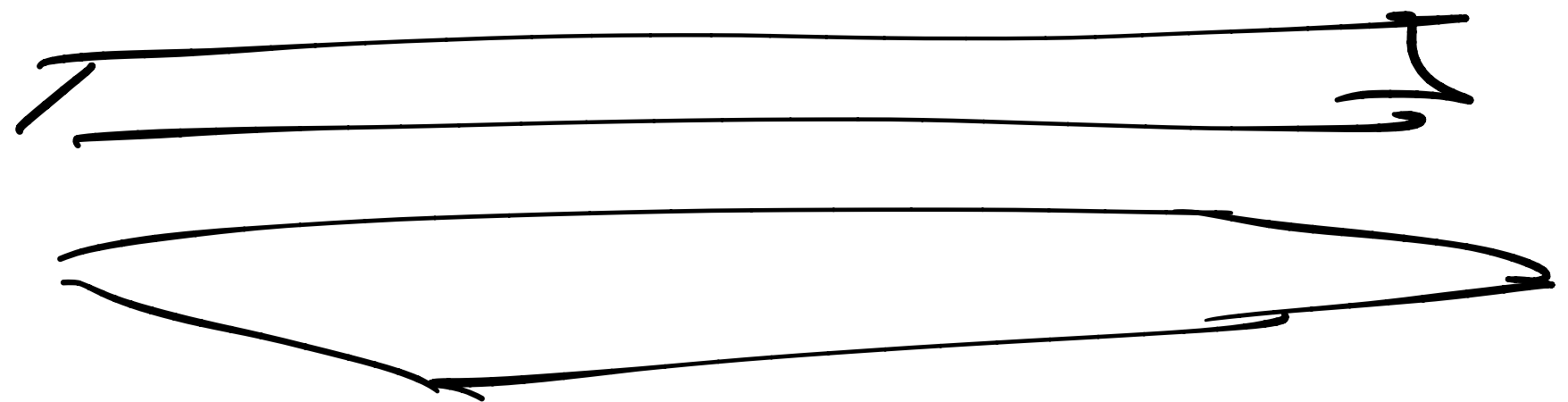
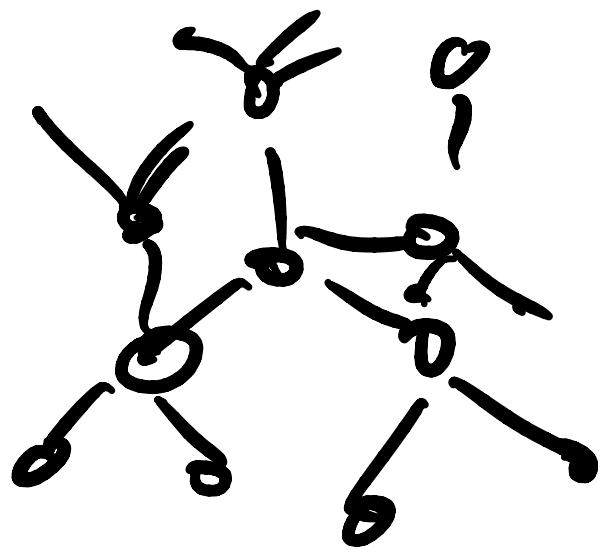
Liquid crystals  
ordered



Differences : density, compressibility  
between heat capacity, conductivity  
phases

Other kind of "phase": different arrangements of  
same atoms

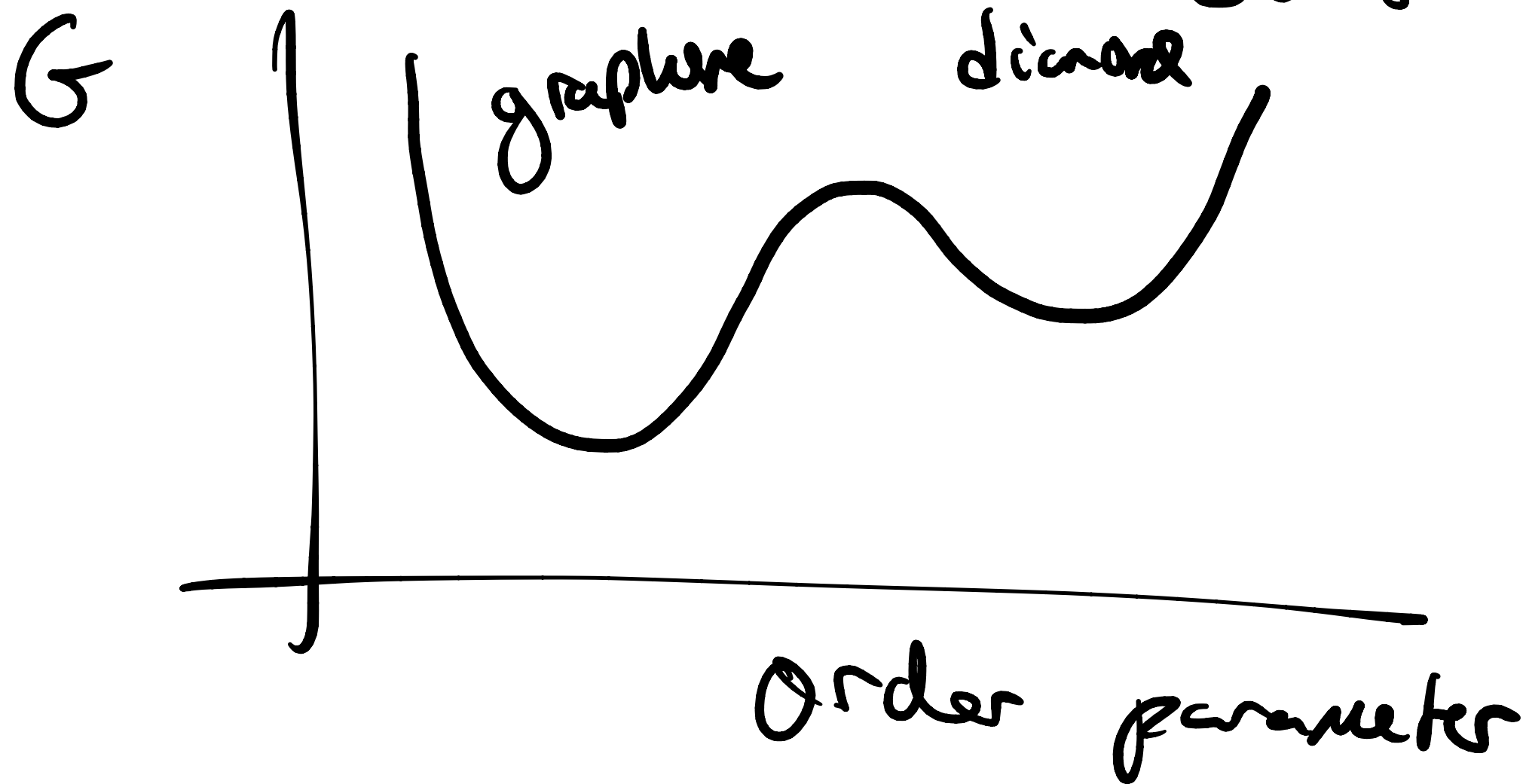
Carbon: Diamond, graphite,  $\rightarrow$  C<sub>60</sub> ...  
 $\rightarrow$  nanotubes  
 $\uparrow$  more stable



(TMD, magic angle graphene)

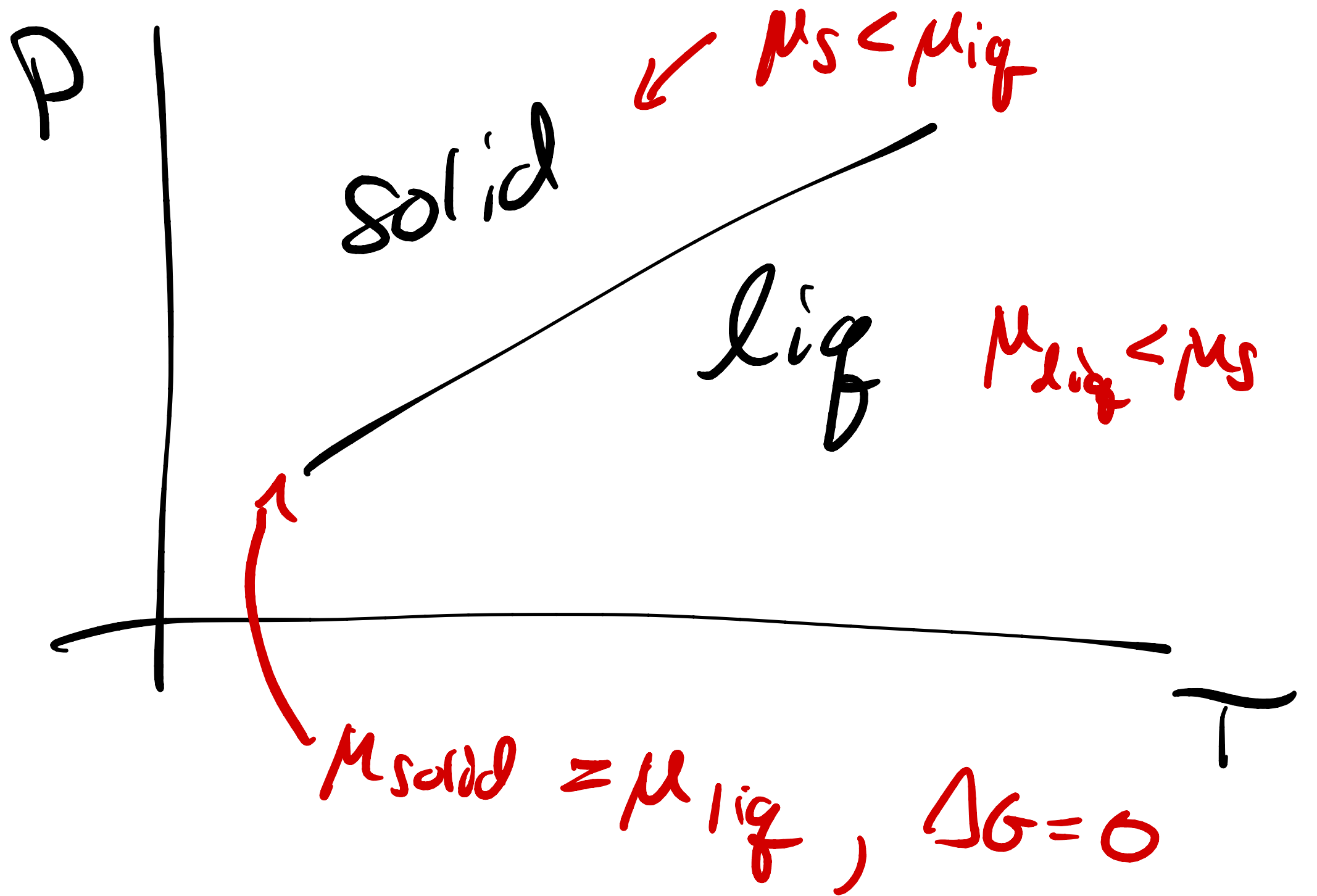
# Stability

system is in a particular phase because  $G$  is lower  
(const  $T$  &  $P$ )





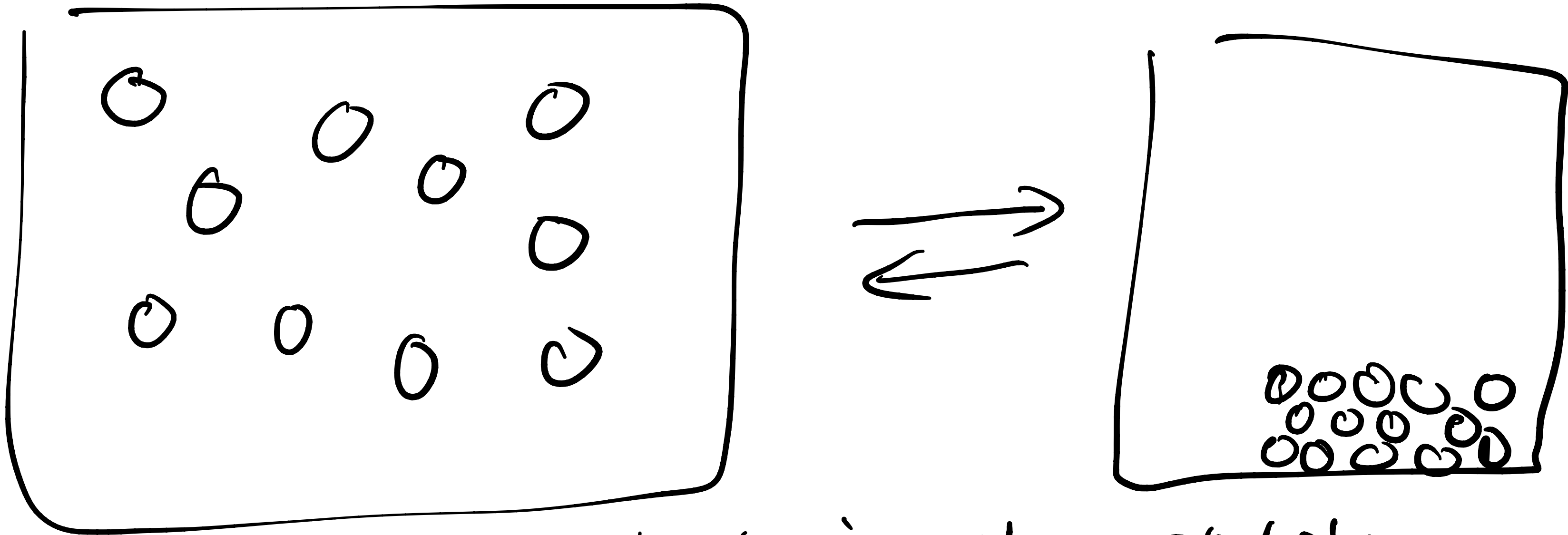
Phase diagram P



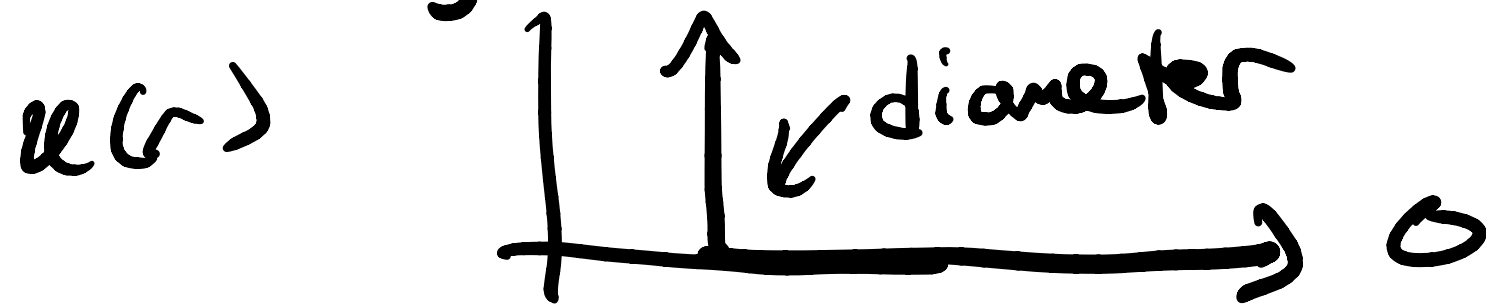
$$G = H - TS$$

$\hookrightarrow \mu = \bar{H} - T\bar{S}$  @ T, P in a certain phase

Old Question: Should hard spheres/disc  
crystallize



Hard: only interaction is volume exclusion



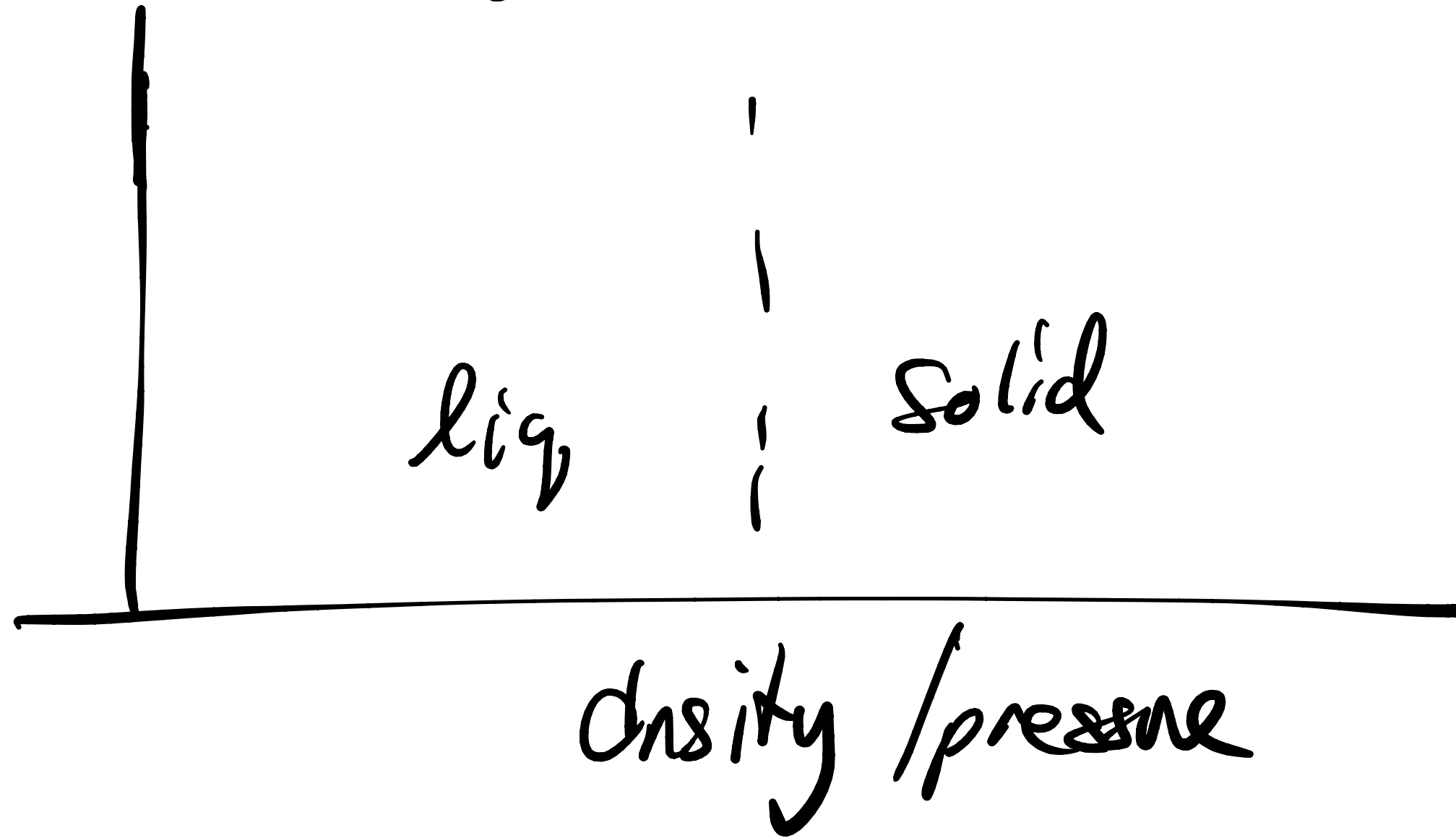
$$A = E - TS$$

for hard spheres,  $E = 0$

$$A = -TS$$

Crystallization means that the  
crystal has lower entropy

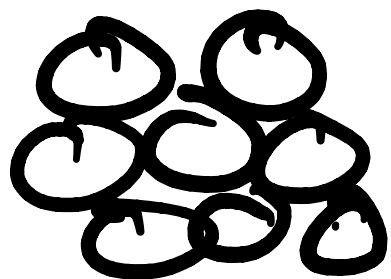
Hard particles crystallize at some density



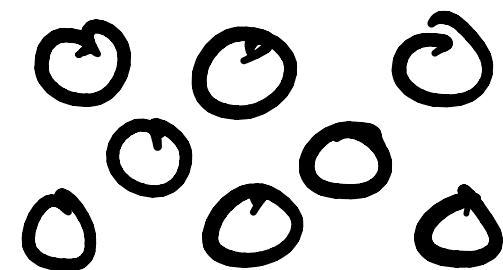
at high density: reach random state

where every thing is touching

at some density



same density



higher entropy

Non-equilibrium "phase"

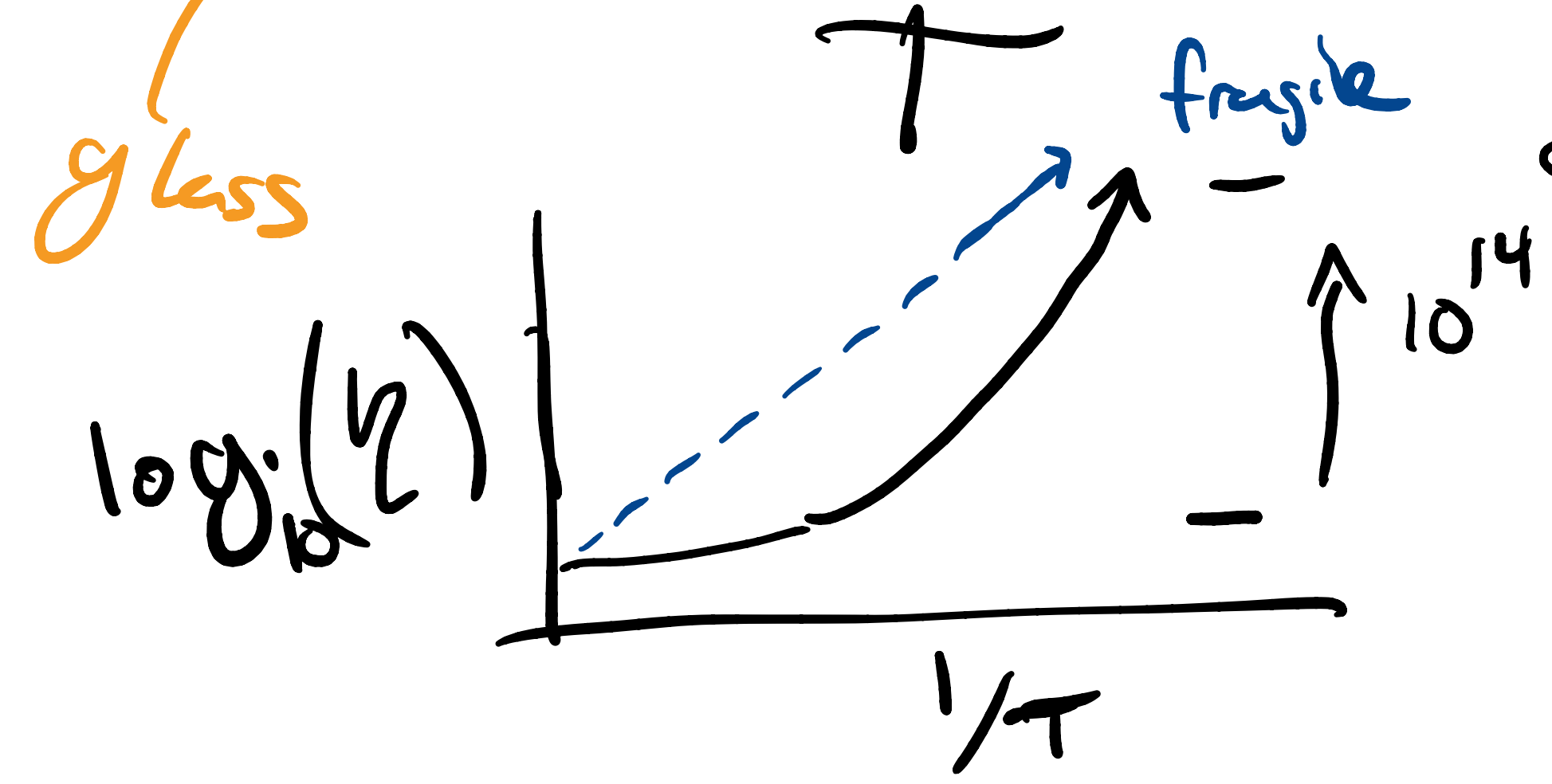
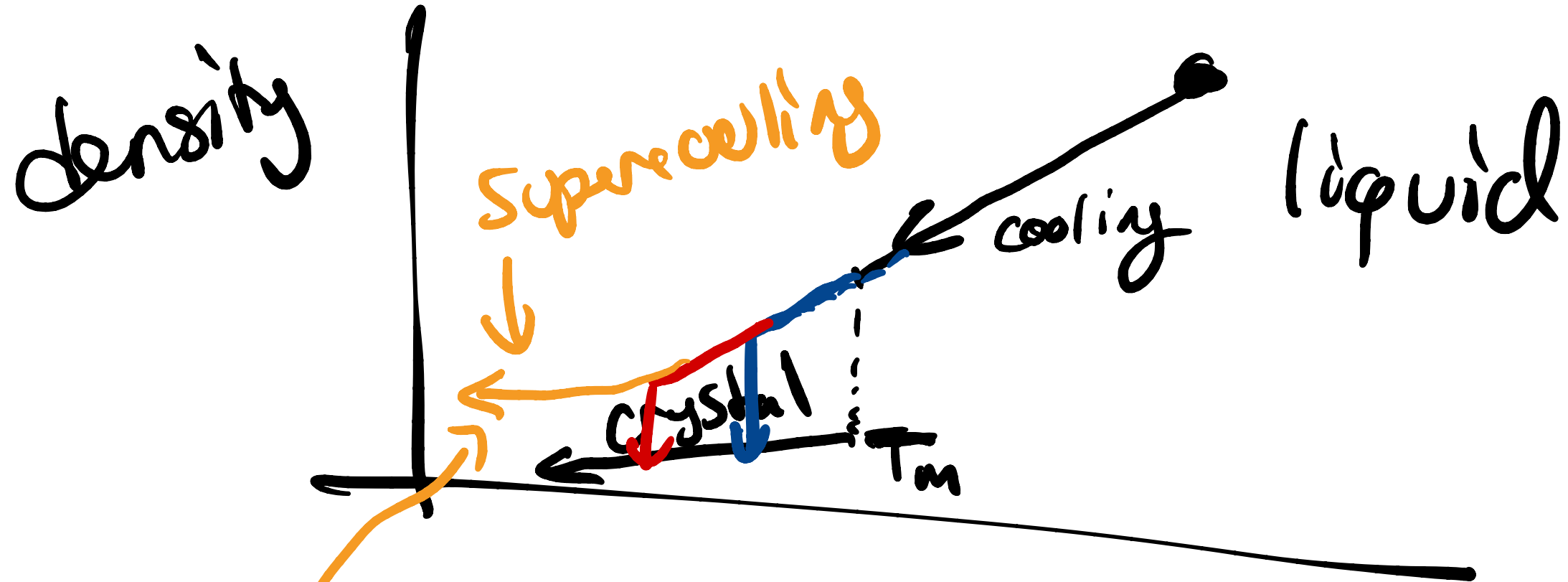
Can get stuck in a configuration  
that is not local free energy minimum

Change conditions "quickly", avoid phase  
change

Supercooled liquid - glass

# Supercooled liquid

glass is still a liquid



$\tau_a \leftarrow$  more liquid  
 100 s  
 $10^{-12}$  s  $\rightarrow$  100 s

glass