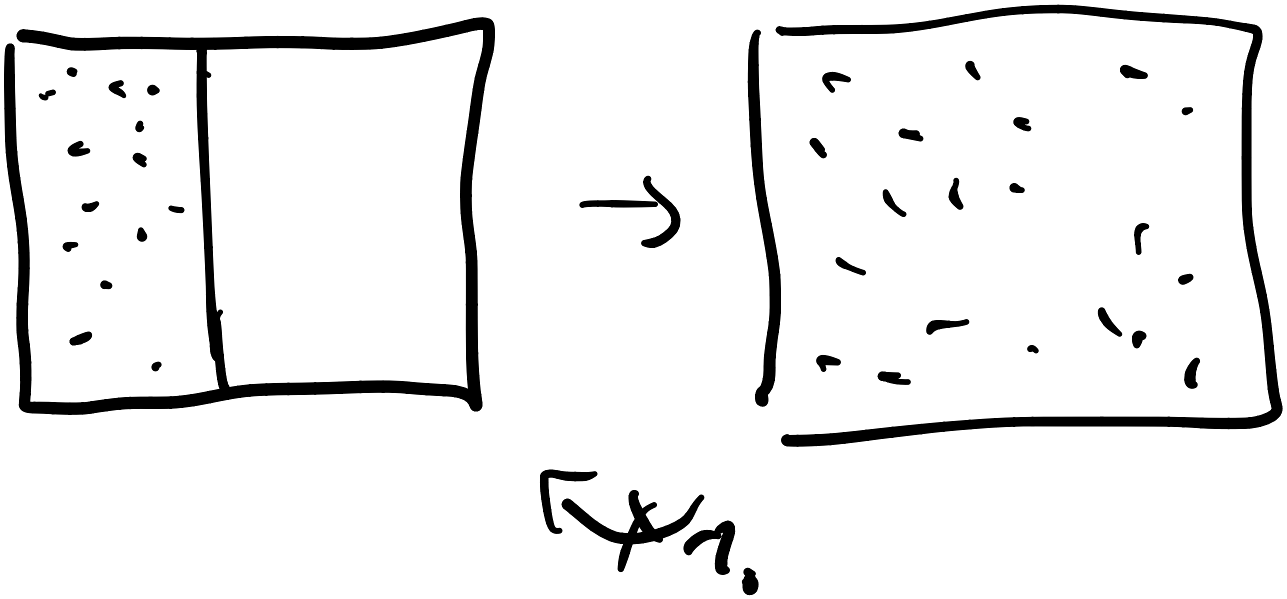
Lecture 7 Carnot cycle / entropy

Entropy

Energy is conserved (in general)



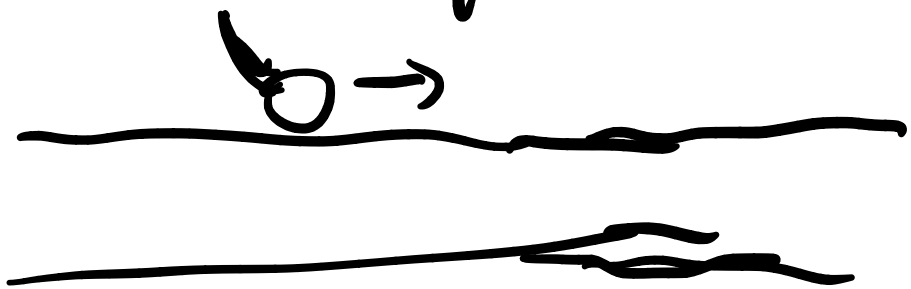
spontaneous process

motion is given from Newton's eqns ($F=ma$)

Idea: concept called entropy
which measures "disorder" system
sets direction of spontaneous processes
going to see change in entropy always ↑ (or 0)
for changes of state

Consider / get to classical entropy
2nd law (Claussius): no process
is possible where q flows from
cold to hot

(counter example:

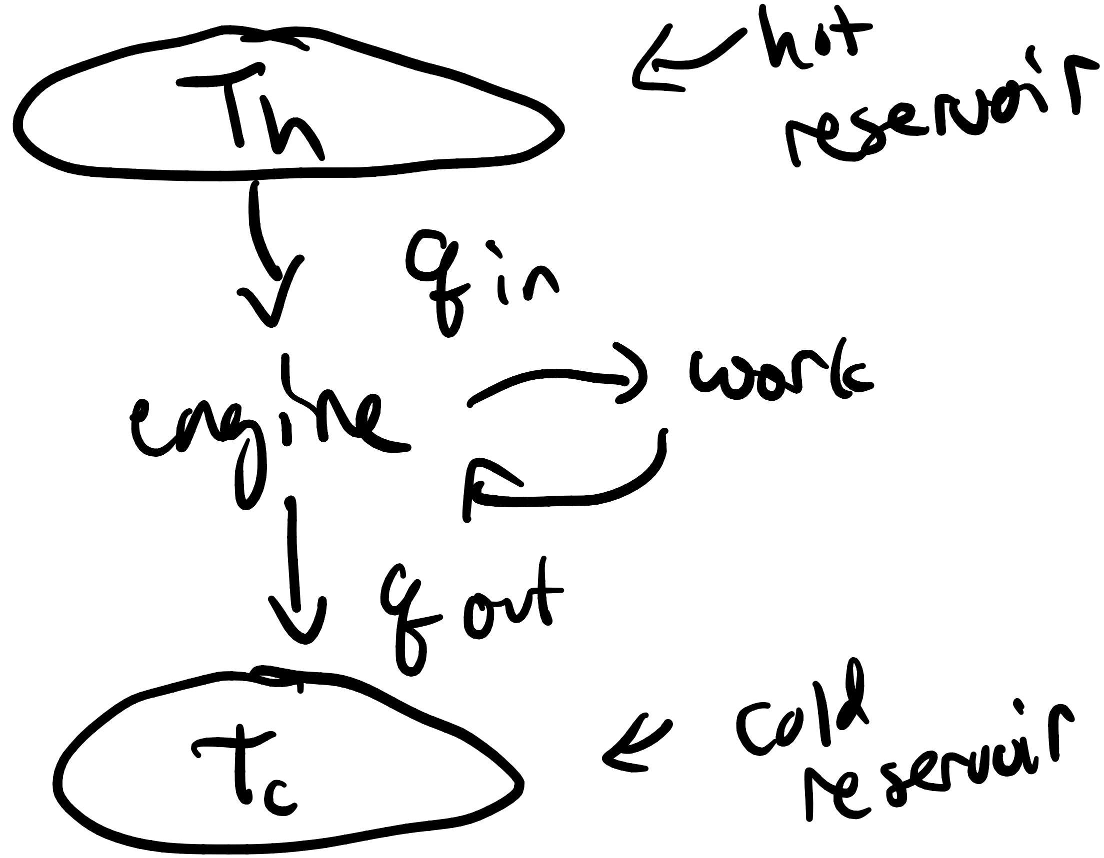


what we know, "backwards processes, negative work"
)

Engine Cycle (ends where it starts)

converts energy from one form to another (create mechanical work)

Diagram as



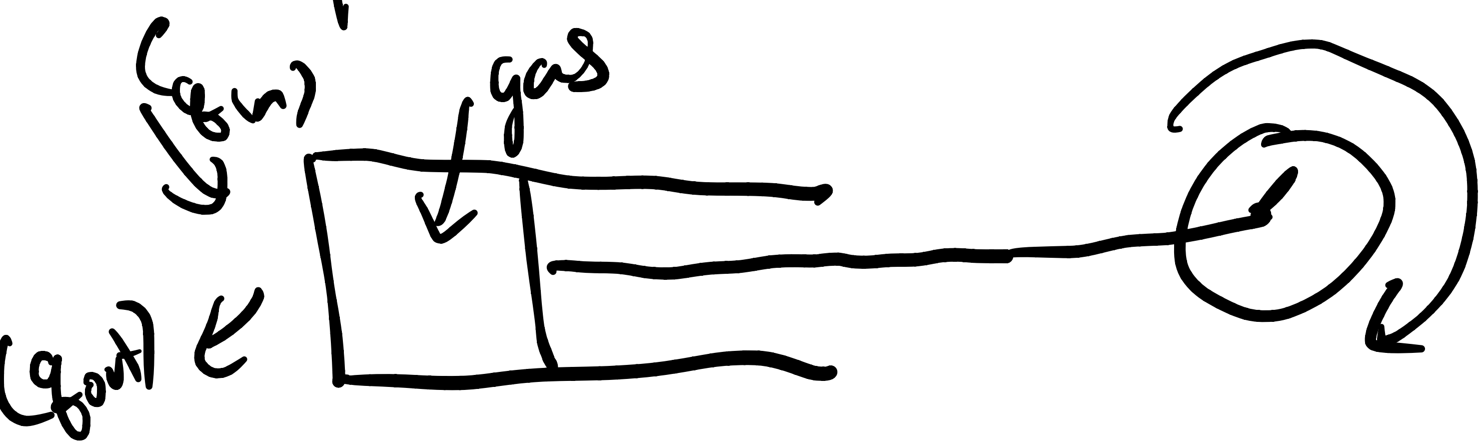
Not all heat can be converted to useful work, lose energy to environment (another version of 2nd law)

Define efficiency

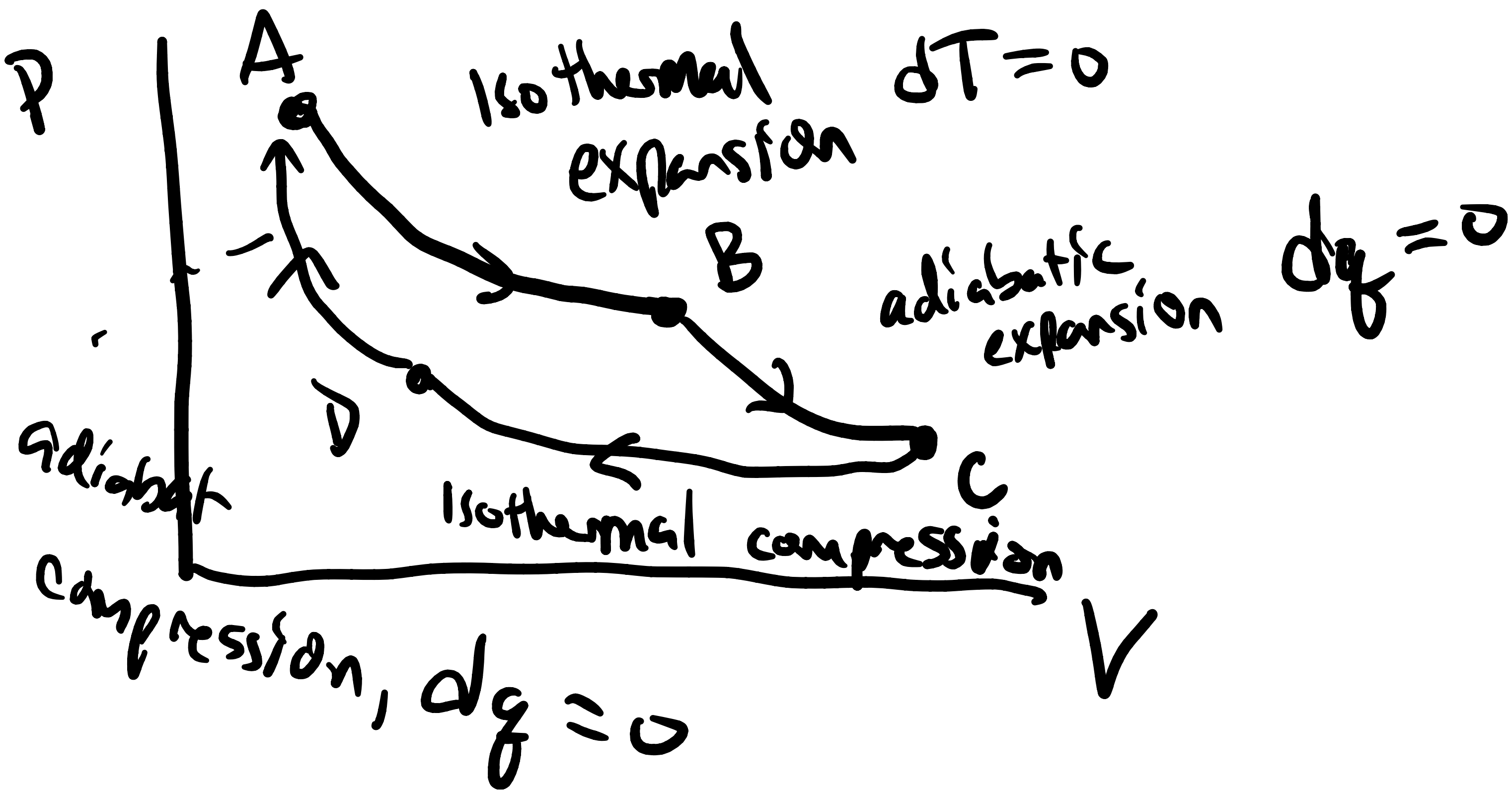
$$\epsilon = \frac{W_{done}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$\Delta E_{cycle} = 0$,

$0 \le \epsilon \le 1$, analyze ϵ for a particular "engine" - Carnot Cycle



4 Steps of Carnot cycle



① A → B, isothermal expansion

connected to hot reservoir, ~~q~~ gas in

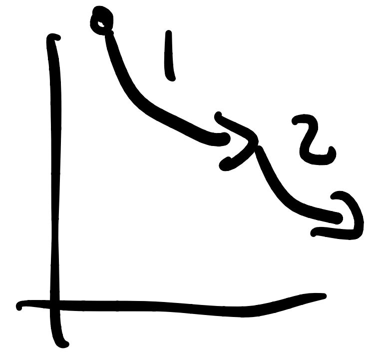
@ $T = T_{hot}$, $dq \uparrow$ work out

(2) adiabatic expansion

system isolated, $dq = 0$

w out

lower the temperature



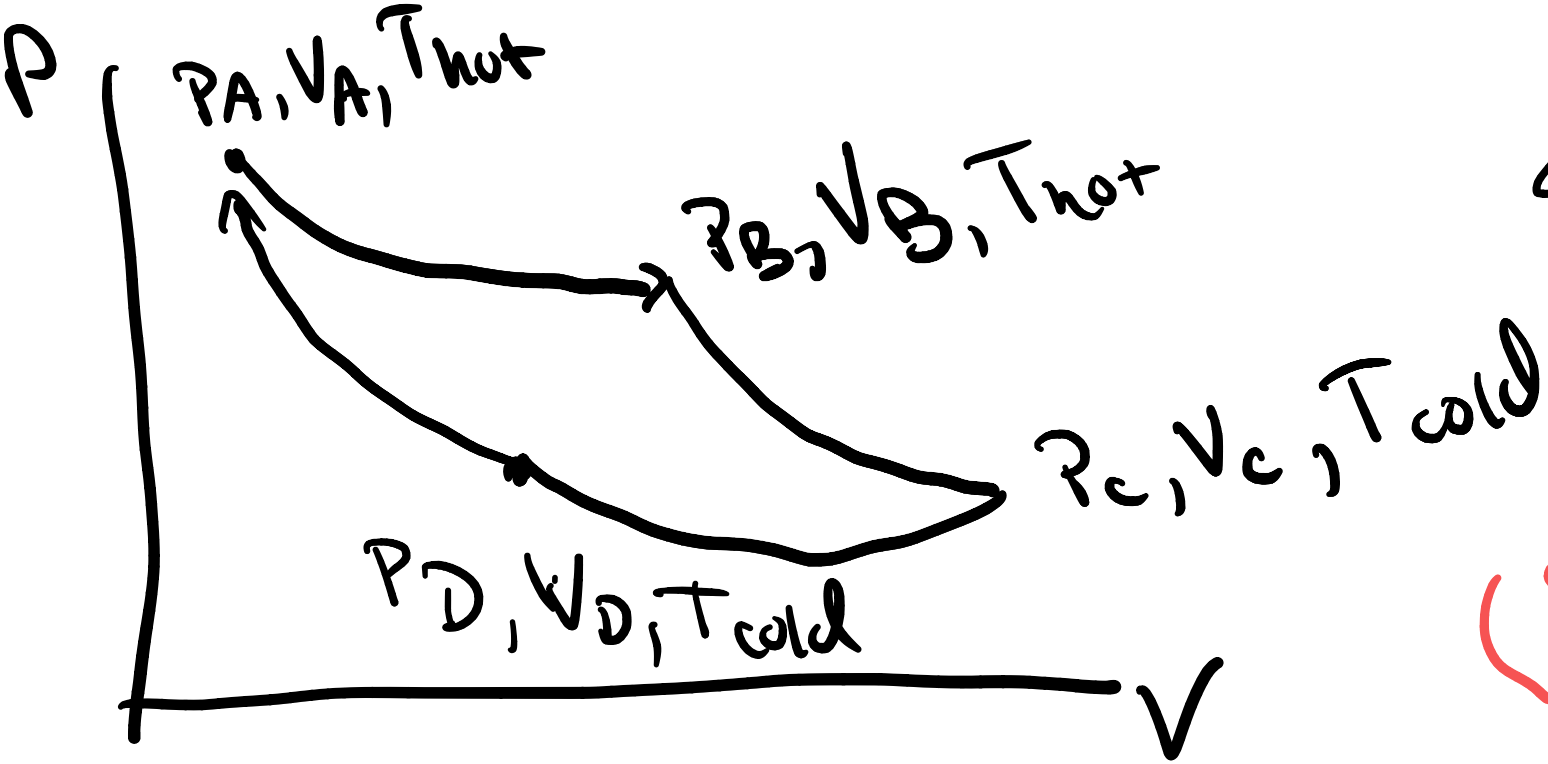
(3) A T_c, connect to cold reservoir

push in plunger $\quad T = T_{cold}$

q out, w in

(4) decouple, push in plunger, T goes up to T_{hot}

$P = P_A$ at end, $dq = 0$, w in



← cycle

(draw graph of T vs V what does that look like?)

For cycle, $\Delta \mathcal{E}_{AB} + \Delta \mathcal{E}_{BC} + \Delta \mathcal{E}_{CD} + \Delta \mathcal{E}_{DA} = 0$

$$W^{done}_{total} = \oint P\, dV > 0 \quad \text{(area)}$$

Want to maximize work, $T_h \uparrow \; T_c \downarrow$

For ideal gas

$dE = dq + dw$ (system)

	flow heat	work done
① isothermal expansion $\Delta E = 0$ $dq = -dw$	$nRT_{hot} \ln\left(V_B/V_A\right)$	$nRT_{hot} \ln\left(V_B/V_A\right)$
② adiabatic $dq = 0$	0	$w = \Delta E = -C_v\left(T_{cold} - T_{hot}\right)$
③ isothermal compression	$nRT_{cold} \ln\left(V_D/V_C\right)$	$nRT_{cold} \ln\left(V_D/V_C\right)$
④ adiabatic compression	0	$-C_v\left(T_{hot} - T_{cold}\right)$

cancel

For cycle $W_{total} = W_1 + W_3$ ← expansion

$$= nRT_{hot} \ln(V_B/V_A)$$

$$+ nRT_{cold} \ln(\underbrace{V_D/V_C}_{V_D < V_A})$$

negative compression

want to maximize T_{hot}, minimize T_{cold}

$$V_C/V_B = (T_{cold}/T_h)^{-C_v/nR}$$

$$V_A/V_D = (T_h/T_{cold})^{-C_v/nR}$$

$$\epsilon = \frac{W_{done}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 + \frac{q_3}{q_1}$$

$$q_3/q_1 = \frac{\cancel{nR}\, T_{cold} \ln(V_D/V_C)}{\cancel{nR}\, T_{hot} \ln(V_B/V_A)}$$

expansion formula

expansion formula

give −1

$$\boxed{\epsilon = 1 - \frac{T_{cold}}{T_{hot}}}$$

shows, want ideal efficiency, T_{cold} has to be 0, K or $T_{hot} \uparrow \infty$

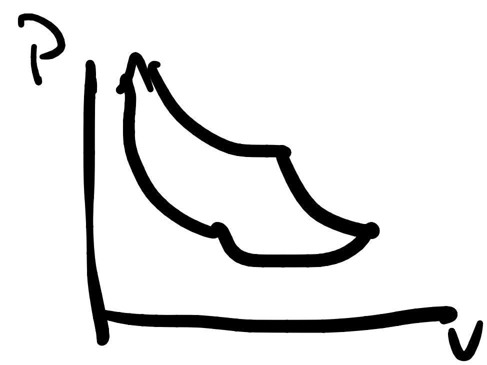
[more general than an ideal gas]

$$q_3/q_1 = -\frac{T_{cold}}{T_{hot}} = -\frac{T_3}{T_1}$$

$$\Rightarrow \quad q_3/T_3 = -q_1/T_1$$

$$\Rightarrow \quad q_1/T_1 + q_3/T_3 = 0$$

exists a quantity which is a state function



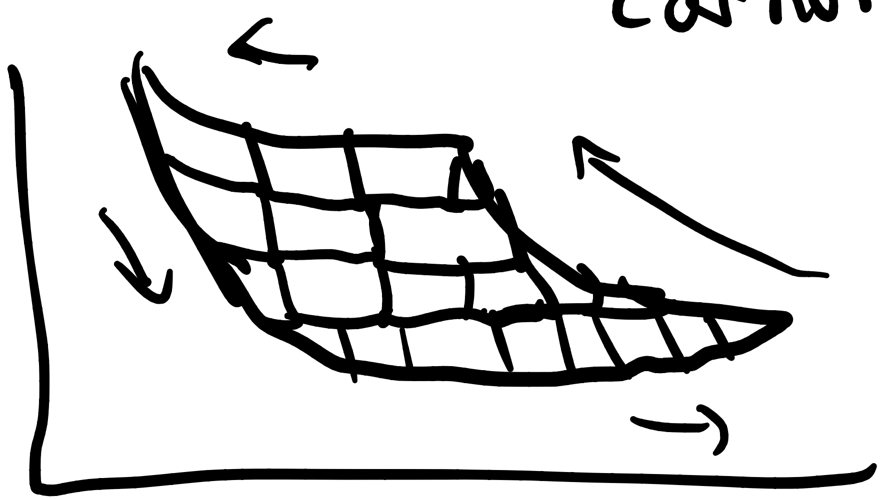
$$\sum_{cycle} q_i^{rev}/T_i = 0 \rightarrow \oint dq/T = 0$$

quantity $S \Leftarrow$ entropy

$$dS = dq^{rev}/T$$, state function

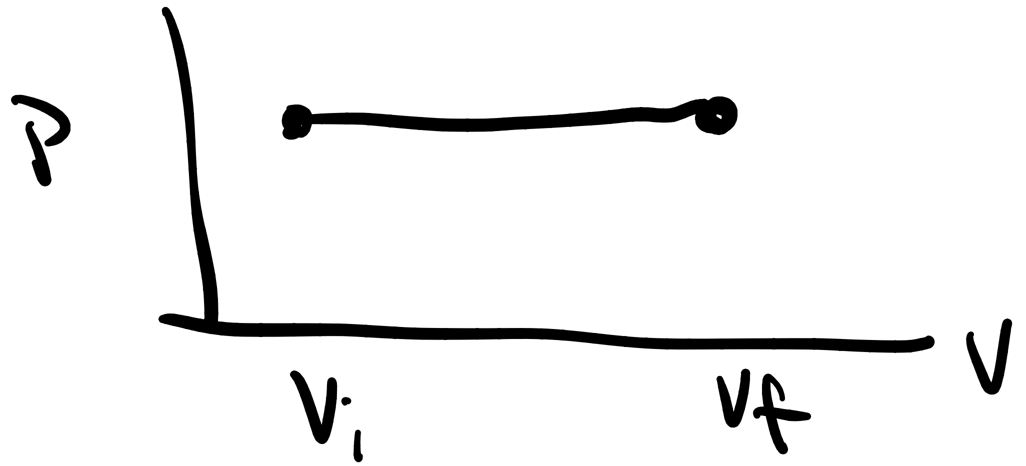
$$\oint dS = 0$$ regardless of path

(see book Any reversible cycle can be made up of a collection of carnot cycles)



Entropy change for reversible processes

(1) Constant P expansion



$$\Delta S = \int \frac{dq^{rev}}{T} = \int_{T_i}^{T_f} \frac{C_P}{T} dT = C_p \ln(T_f/T_i)$$

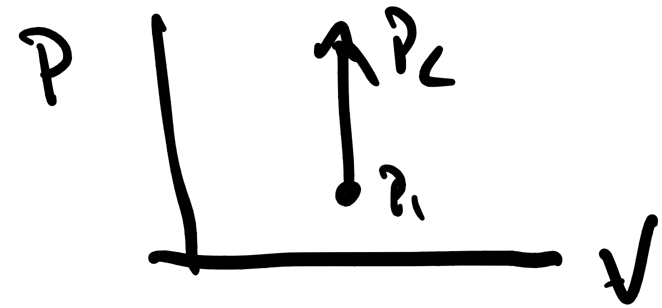
$dq = C\,dT$

ideal gas $P \propto T$

$$= C_p \ln(V_f/T_i)$$

(2) Const volume

$\Delta S = C_V \ln \left(T_2 / T_1 \right)$



(3) Constant T,

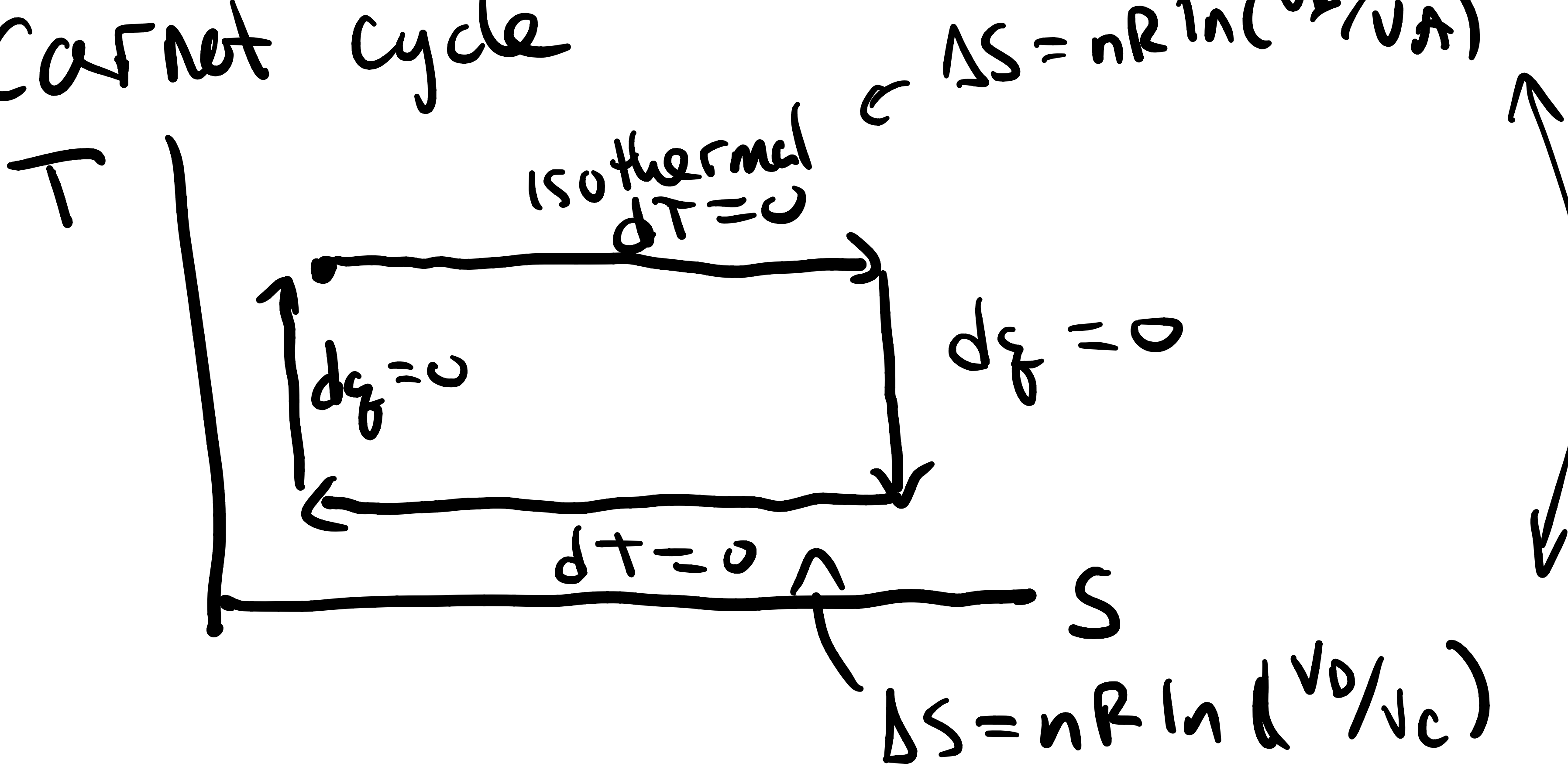
for ideal gas, $dE = 0$, $dq = -dw$
$= PdV$

$$\Delta S = \int_i^f \frac{dq}{T} = \int_{V_1}^{V_2} \frac{P}{T} dV = nR \ln \left(V_2 / V_1 \right)$$

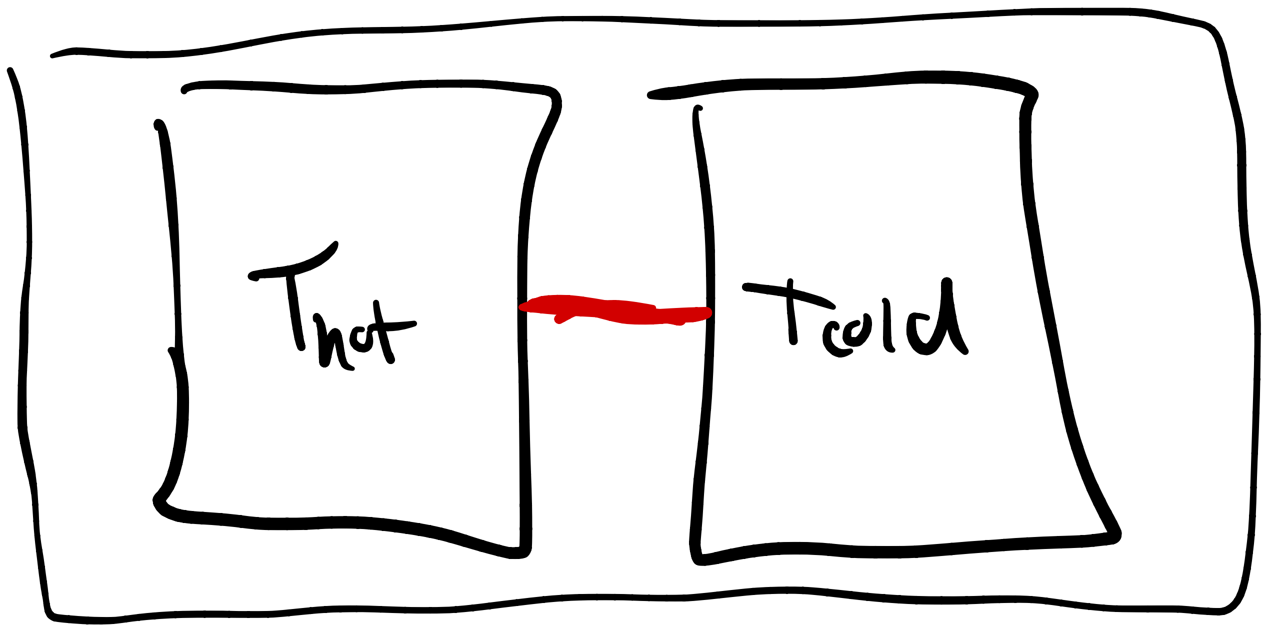
$PV = nRT$, $P/T = \frac{nR}{V}$

(4) adiabat, $dq = 0$, $\Delta S = 0$

Carnot cycle



Instead of a cycle, entropy change when heat flows



isolated

$$\mathcal{E}_{total} = \mathcal{E}_{left} + \mathcal{E}_{right}$$

const volume

$$d\mathcal{E} = d\mathcal{E}_{left} + d\mathcal{E}_{right}$$

$$= dq_{left} + dq_{right} = 0$$