Lecture 6 - Work & Carnot Cycle

How can you change states

1. Constant pressure $dP = 0$
2. Constant volume $dV = 0$
3. Constant temperature $dT = 0$
4. "adiabatic" $dq = 0$

$$dq = C\,dT$$

$$C_v = \left(\partial q / \partial T\right)_v \qquad C_p = \left(\partial q / \partial T\right)_p$$

1st law:

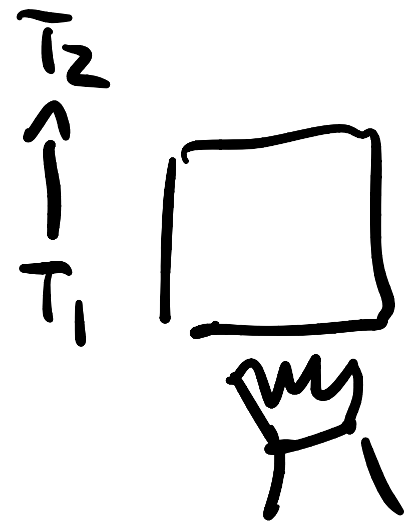
$$dE = \text{đ}q + \text{đ}w$$
$$= \text{đ}q - PdV$$

$\underline{C_p}$ vs $\underline{C_v}$

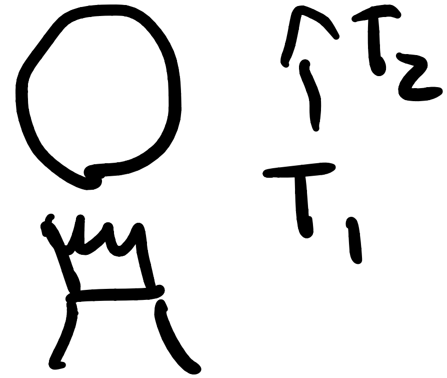
@ const volume, $dV = 0$

$$dE = \text{đ}q = C\,dT$$

$$\Delta E = C\,(T_2 - T_1)$$



Constant pressure



$$dE = \bar{d}q - PdV$$

$$= C_p\, dT - PdV$$

$$\Delta E = C_p \Delta T - P\Delta V \qquad \text{(new } V \,\&\, T\text{)}$$

$$C_v = \left(\partial q/\partial T\right)_v = \left(\frac{\partial E}{\partial T}\right)_v$$

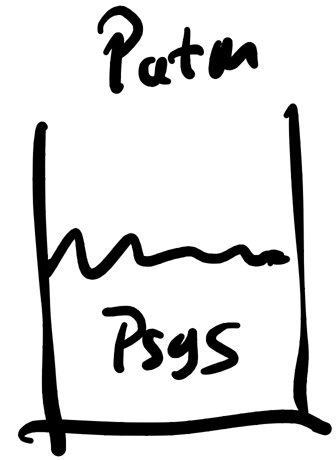
Const P, $\bar{d}q = dE + PdV$

$$C_p = \left(\frac{\partial E + P\partial V}{\partial T}\right)_P \Leftrightarrow \quad \text{define } H = E + PV$$

$$H = \mathcal{E} + PV$$

$$dH = d\mathcal{E} + PdV + \cancelto{0}{VdP}$$

$$= \bar{d}q$$



$$C_p = \left(\partial H / \partial T\right)_P$$

Consider ideal gas

① $PV = nRT$

② $\mathcal{E} = \frac{d}{2} nRT$ $\quad d = 3$ (monotonic)

① $PV = nRT$

(mono)

② $\varepsilon = \frac{3}{2} nRT$

$$C_V = \left(\frac{\partial \varepsilon}{\partial T}\right)_V = \frac{3}{2} nR \quad [\text{constant}]$$

$$C_p = \left(\frac{\partial H}{\partial T}\right)_P$$

$$H = \varepsilon + PV = \left(\frac{3}{2} nRT\right) + (nRT) = \frac{5}{2} nRT$$

$$C_p = \frac{5}{2} nR$$

$C_p > C_V$

$C_p - C_V = nR$

How can you change states

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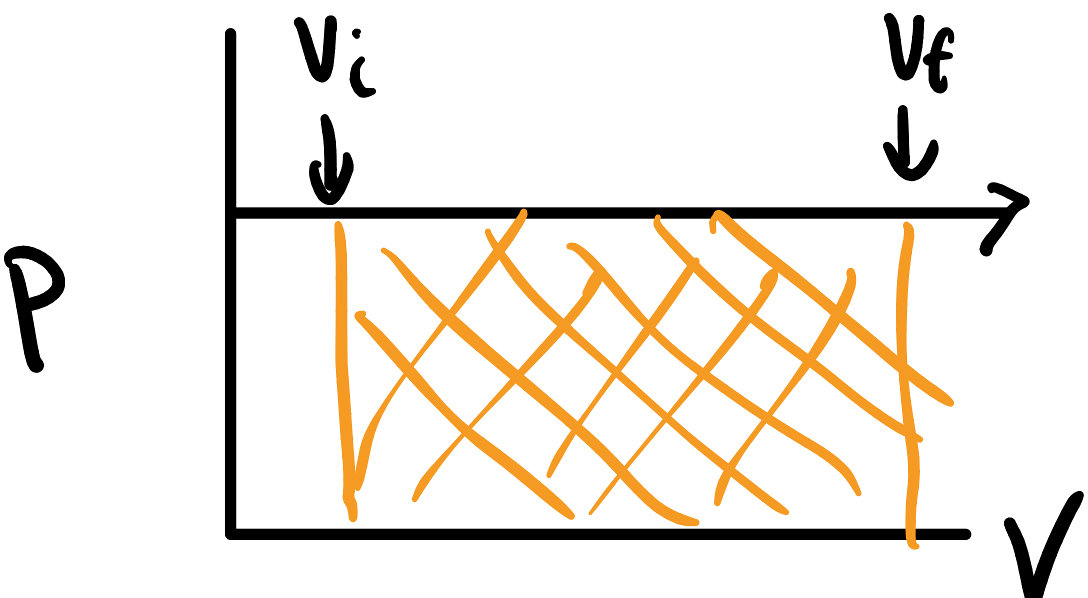
ideal gas

$\left(T = \frac{PV}{nR} \right.$

1 Constant P



work

$$= \int_{V_i}^{V_f} -P\,dV$$

$$= -P(V_f - V_i)$$

$$\Delta E = C_p \Delta T - P\Delta V$$

How can you change states

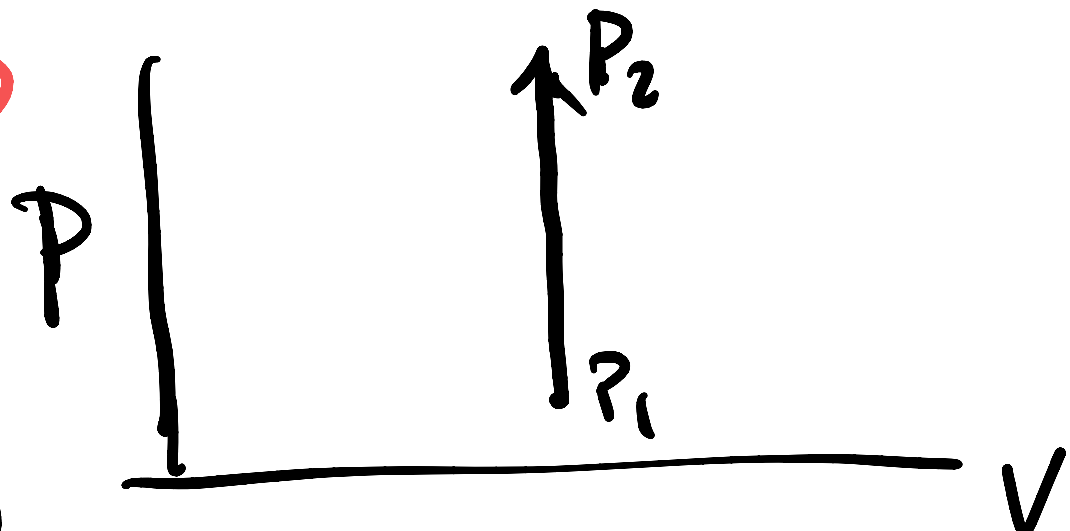
① Constant pressure $dP = 0$

② Constant volume $dV = 0$

③ constant temperature $dT = 0$

④ "adiabatic" $dq = 0$

$dV = 0$



$\Rightarrow dW = -PdV$
$= 0$

go $P_1 \to P_2$

No work!

$$\Delta E = C_v \Delta T$$

How can you change states

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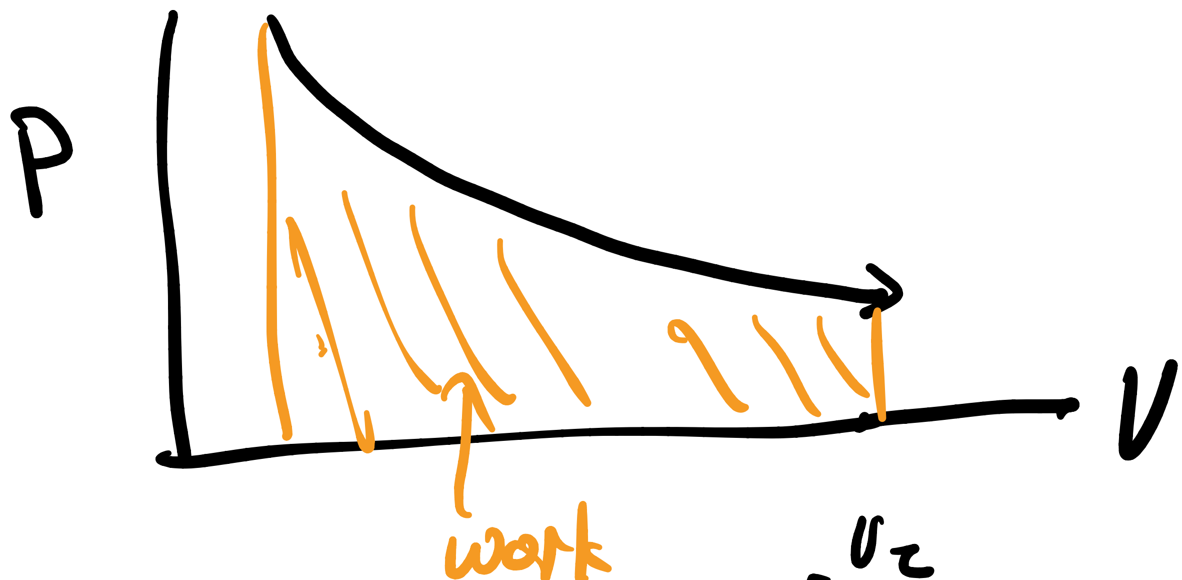
$dT=0$



$$P=\frac{nRT}{V}$$

$$W=-\int_{V_1}^{V_2}\frac{nRT}{V}\,dV=-nRT\int_{V_1}^{V_2}\frac{1}{V}\,dV$$

$$=-nRT\ln\left(V_f/V_i\right)$$

$$d\mathcal{E} = đq + đw$$

ideal gas

$$\mathcal{E} = \frac{3}{2} nRT$$

$$\Rightarrow đq = -đw$$

$$d\mathcal{E} = \frac{3}{2} nR\underbrace{dT}_{=0}$$

$$= 0$$

$$q = +nRT\ln(V_2/V_1)$$

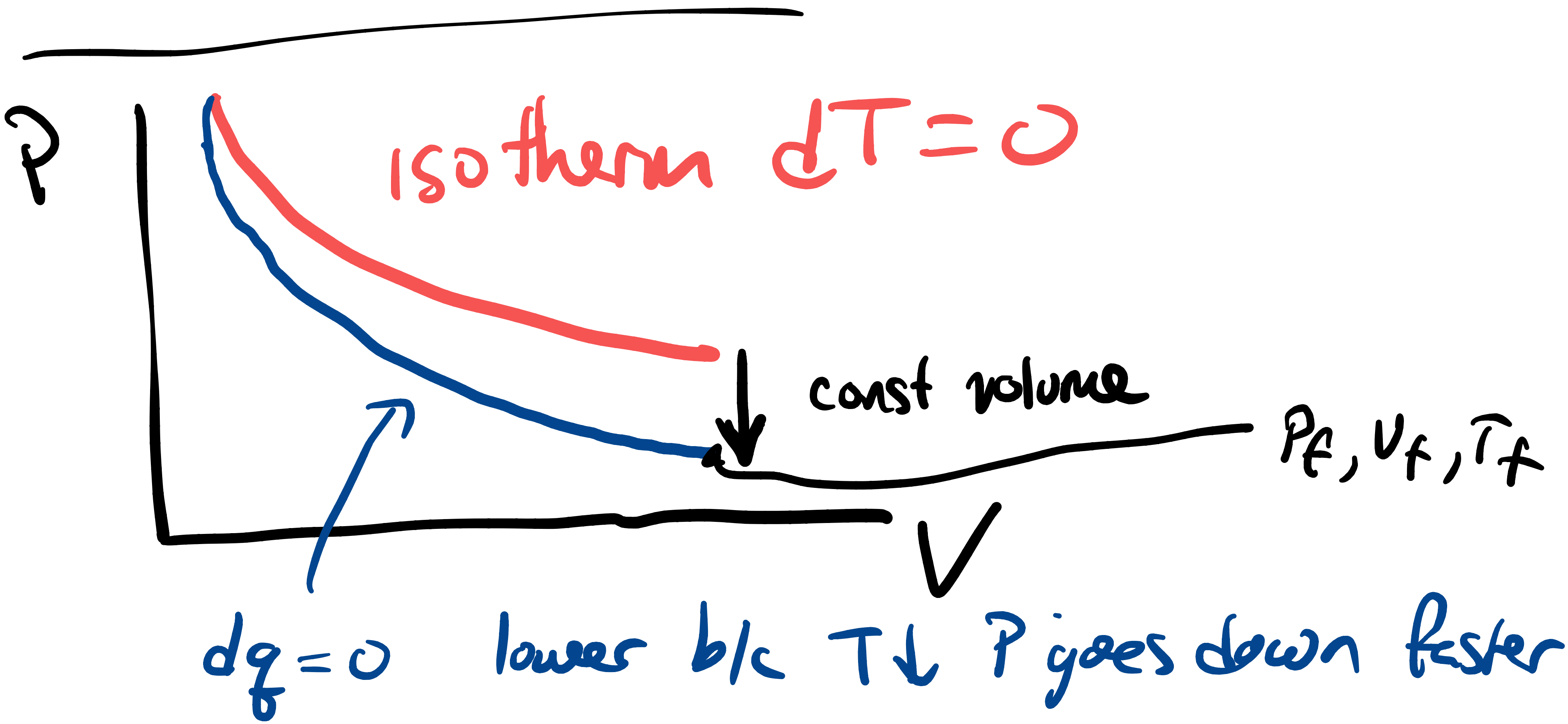
if expand, $V_2 > V_1$, $q > 0$

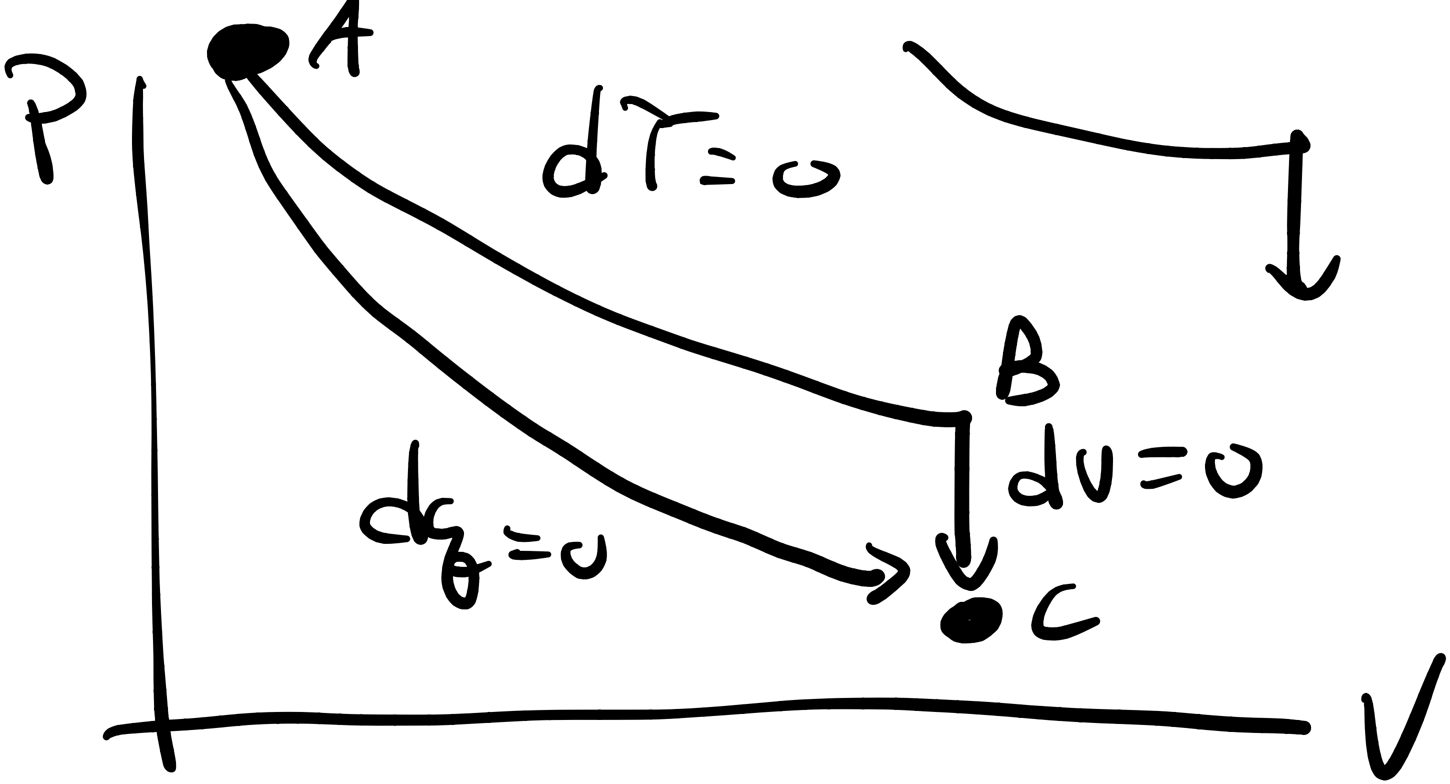
q goes in

④ $đq$, adiabatic expansion/compression

$$dE = \cancel{đq}^{\,0} + đw$$
$$= -P \quad dV$$
$$= -\frac{nRT(V)}{V} dV \qquad \text{(ideal gas)}$$

Expansion leads to cooling in this case


P
isotherm dT = 0
const volume
P_f, V_f, T_f
V
dq = 0
lower b/c T↓ P goes down faster



(ideal gas)

$$\Delta \mathcal{E} = \Delta \mathcal{E}_{AB} + \Delta \mathcal{E}_{BC}$$

$$= 0 + C_v \Delta T = \Delta \mathcal{E}_{AC}$$

$$C_v dT = \frac{nRT}{V} dV \Rightarrow C_v \frac{dT}{T} = \frac{nR}{V} dV$$

$$C_v \frac{1}{T} dT = -\frac{nR}{V} dV$$

$$C_v \ln\left(T_f / T_i\right) = -nR \ln\left(V_f / V_i\right)$$

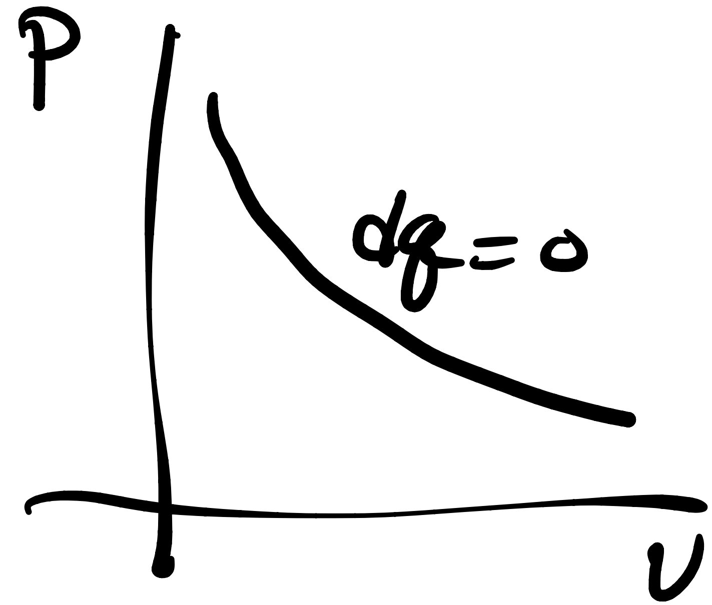
$$\ln\left(T_f / T_i\right) = -\frac{nR}{C_v} \ln\left(V_f / V_i\right)$$

$$T_f / T_i = \left(V_f / V_i\right)^{-nR/C_v}$$

$\Rightarrow$ expansive cooling

$$T_f / T_i = \left(V_f / V_i \right)^{-nR/C_v}$$

$$T = \frac{PV}{nR}$$



$$\frac{P_f V_f}{P_i V_i} = \left(V_f / V_i \right)^{-nR/C_v} = \left(V_i / V_f \right)^{nR/C_v}$$

$$P_f / P_i = \left(V_i / V_f \right)^{\frac{nR}{C_v} + 1}$$

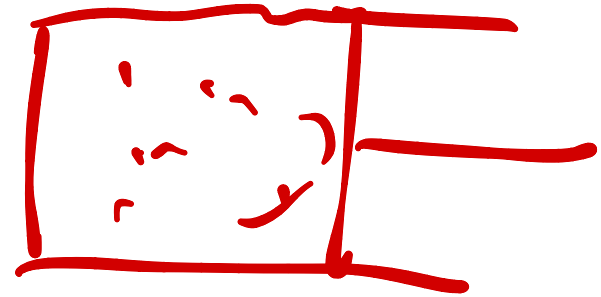
$$= \left(V_i / V_f \right)^{C_p / C_v}$$

$$\left[\frac{nR}{C_v} + 1 = \frac{nR + C_v}{C_v} \right.$$

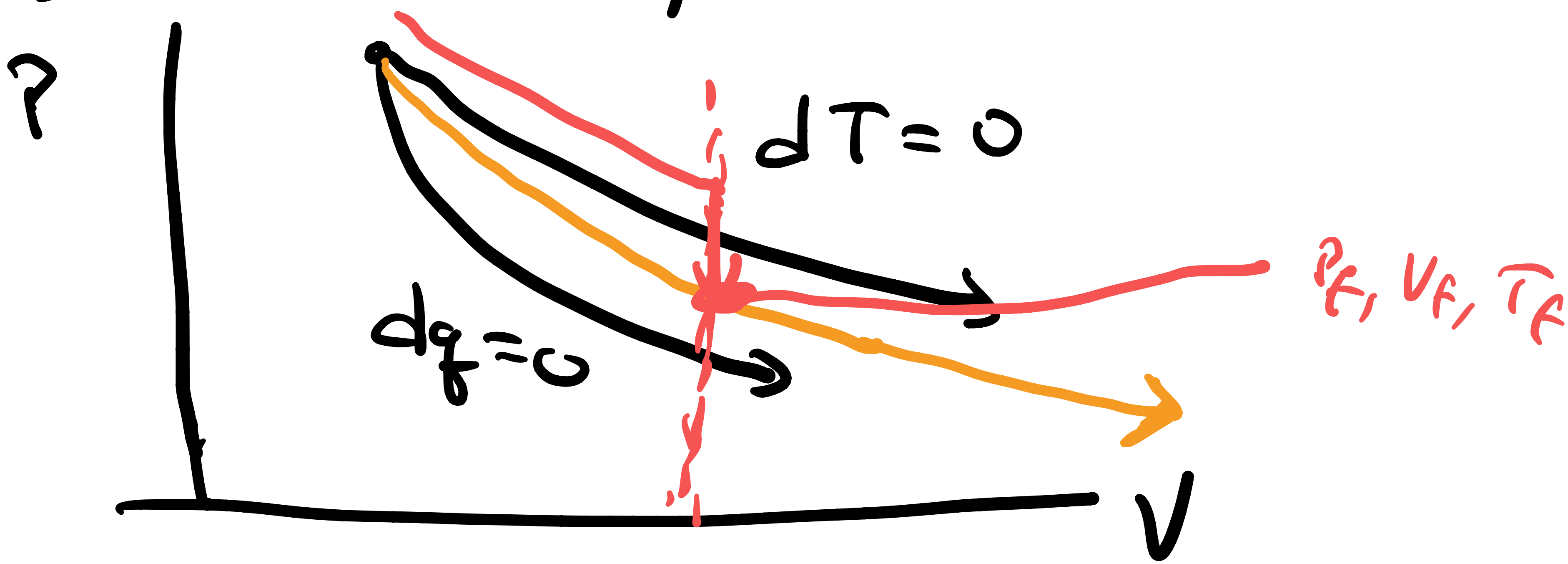
$$W = -\int_{V_i}^{V_f} P\,dV = \ldots$$

$$P(V) = P_i\left(\frac{V_i}{V}\right)^{C_P/C_V}$$

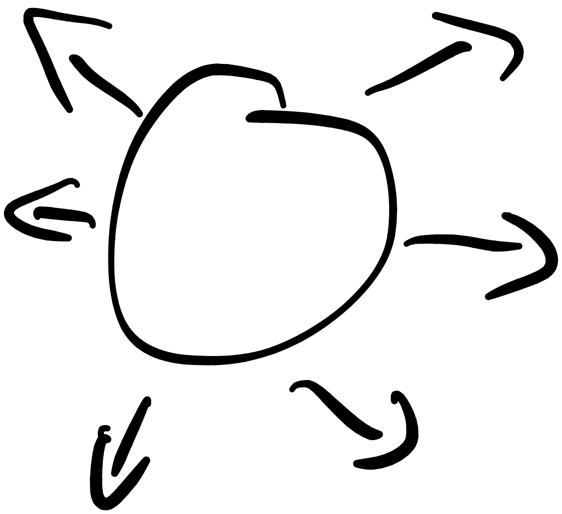
$P_{in} > P_{out}$

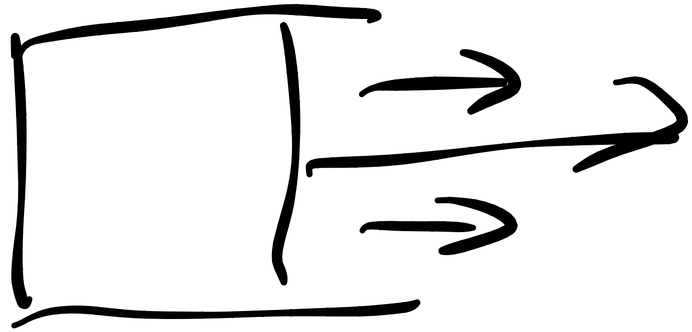


Irreversible expansions



Expansion against a vacuum





$P_{out} = 0$

$$w = -\int P\,dV = 0$$

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