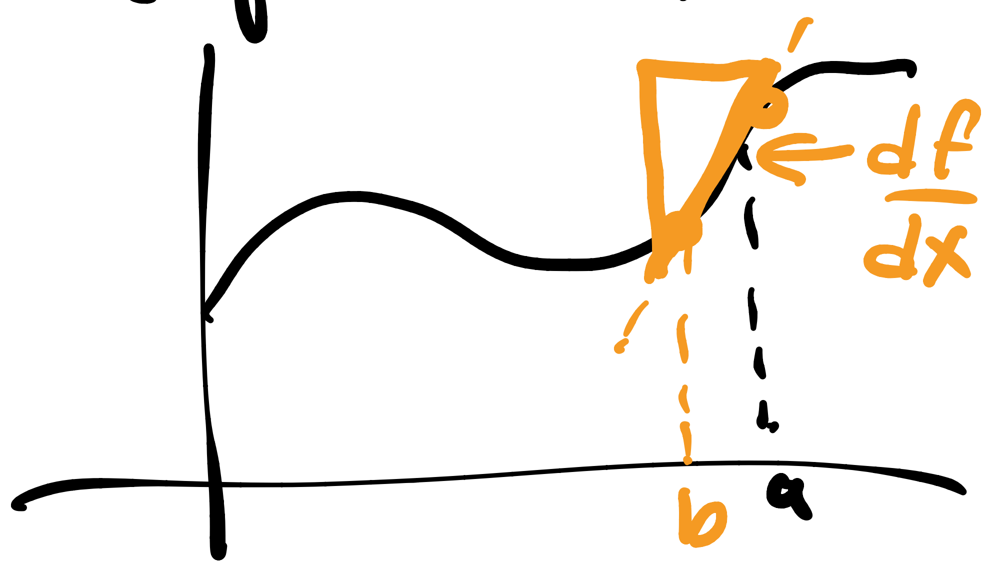
Lecture 4 Calc review + thermo intro

(1) derivative – rate/slope of a function at a point

$f(x)$ function

$\left.\frac{df}{dx}\right|_{x=a}$ is slope at $x=a$

$$\frac{f(b)-f(a)}{b-a}$$



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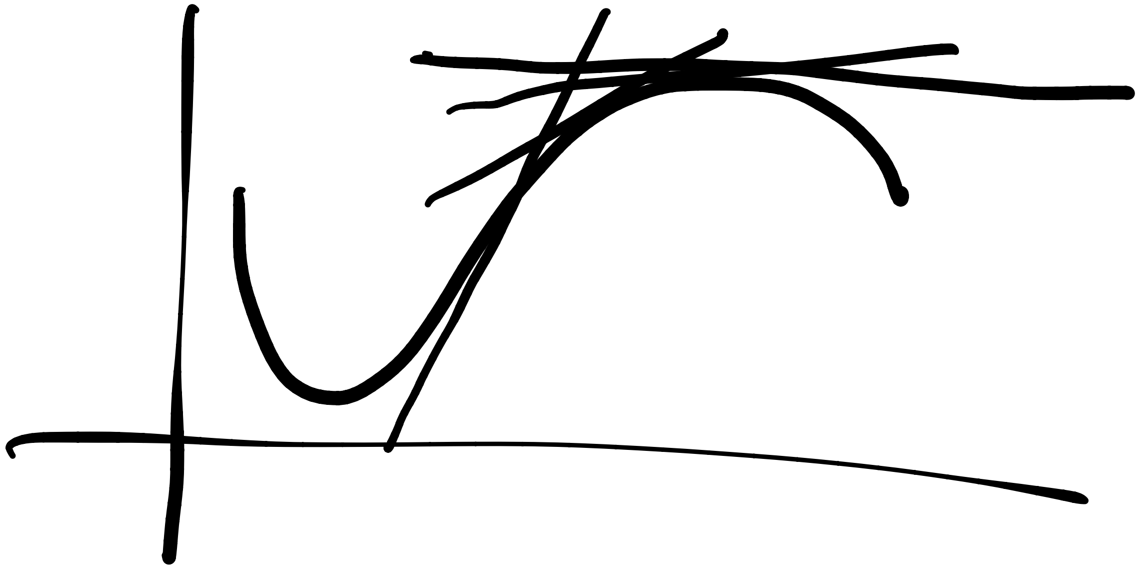
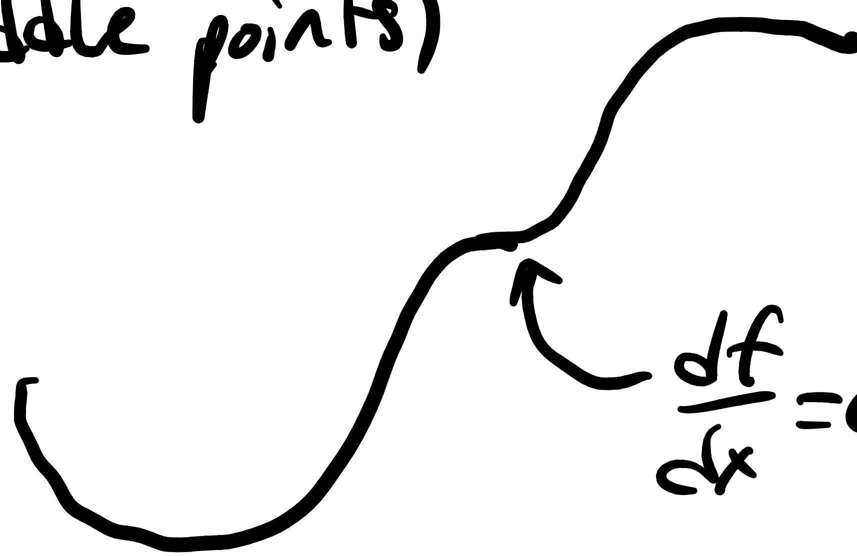


$\leftarrow \frac{df}{dx} = 0$

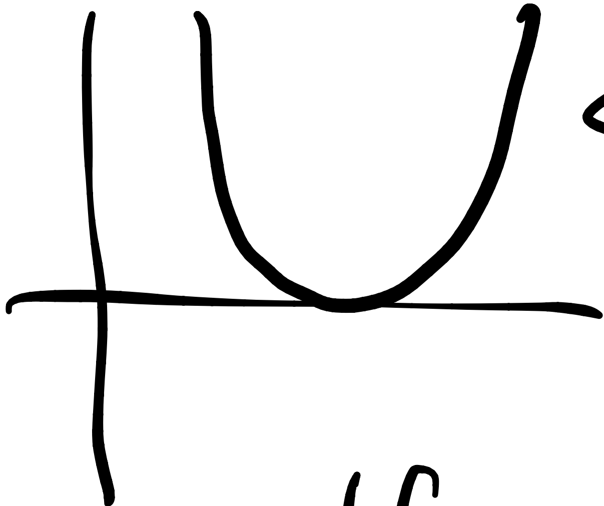
Max/min happen when $df/dx = 0$

(saddle points)



$\frac{d^2f}{dx^2} = ?$

$\frac{df}{dx} = 0$ saddle

 $\longleftrightarrow f(x) = x^2$

$f(x) = -x^2$

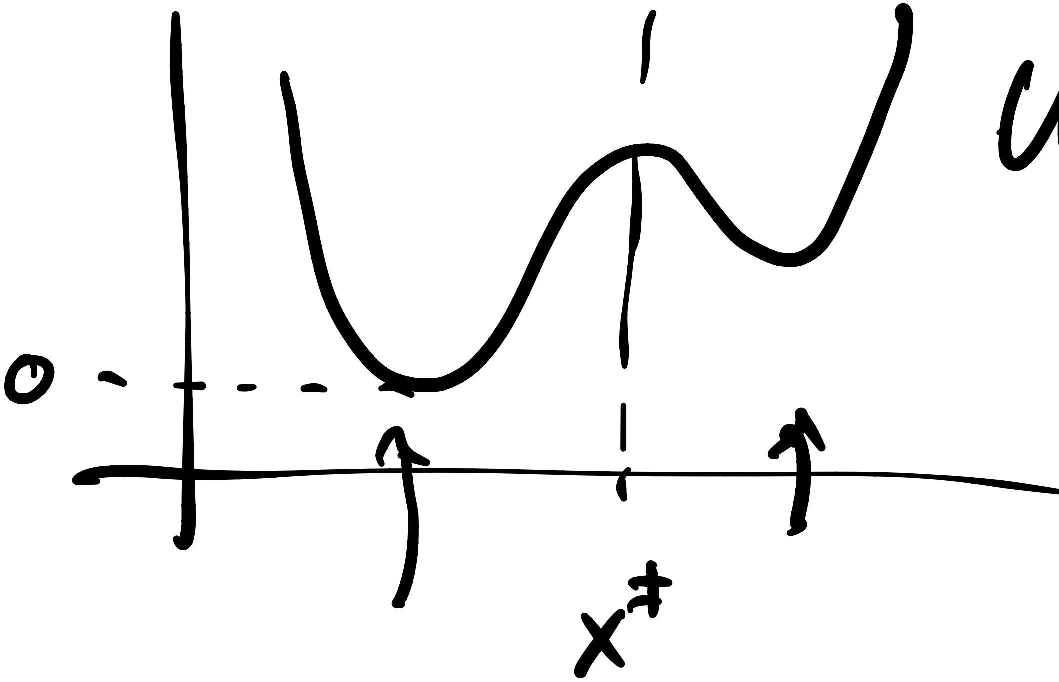
$\frac{df}{dx} = 2x \qquad \frac{d}{dx}\left(\frac{df}{dx}\right) = 2 > 0$

2nd derivative $> 0 \Rightarrow$ min

$< 0 \Rightarrow$ max

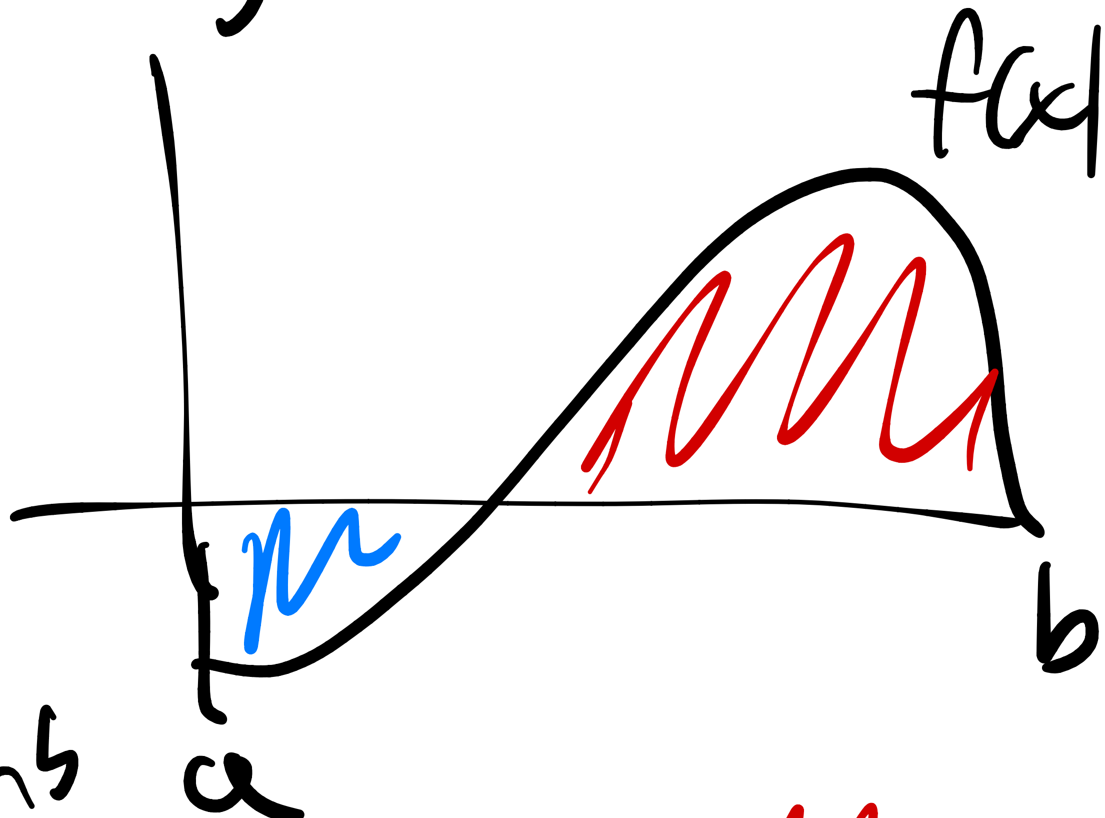
$f(x) = x^n \qquad f' = nx^{n-1}$

$de^x/dx = e^x \qquad d\ \ln(x)/dx = 1/x$



$$U(x) \approx \begin{cases} \frac{1}{2} k_1 (x - x_0)^2 & x \ll x^{\ddagger} \\ \left(\frac{1}{2} k_2 (x - x_1)^2 + a\right) & x \gg x^{*} \\ -\frac{1}{2} k_3 (x - x^{\ddagger}) + U(x^{\ddagger}) & x \sim x^{\ddagger} \end{cases}$$

(3) Integrals



$$\int_a^b f(x)\,dx = \quad - \quad$$

(4) Integrals - anti derivative

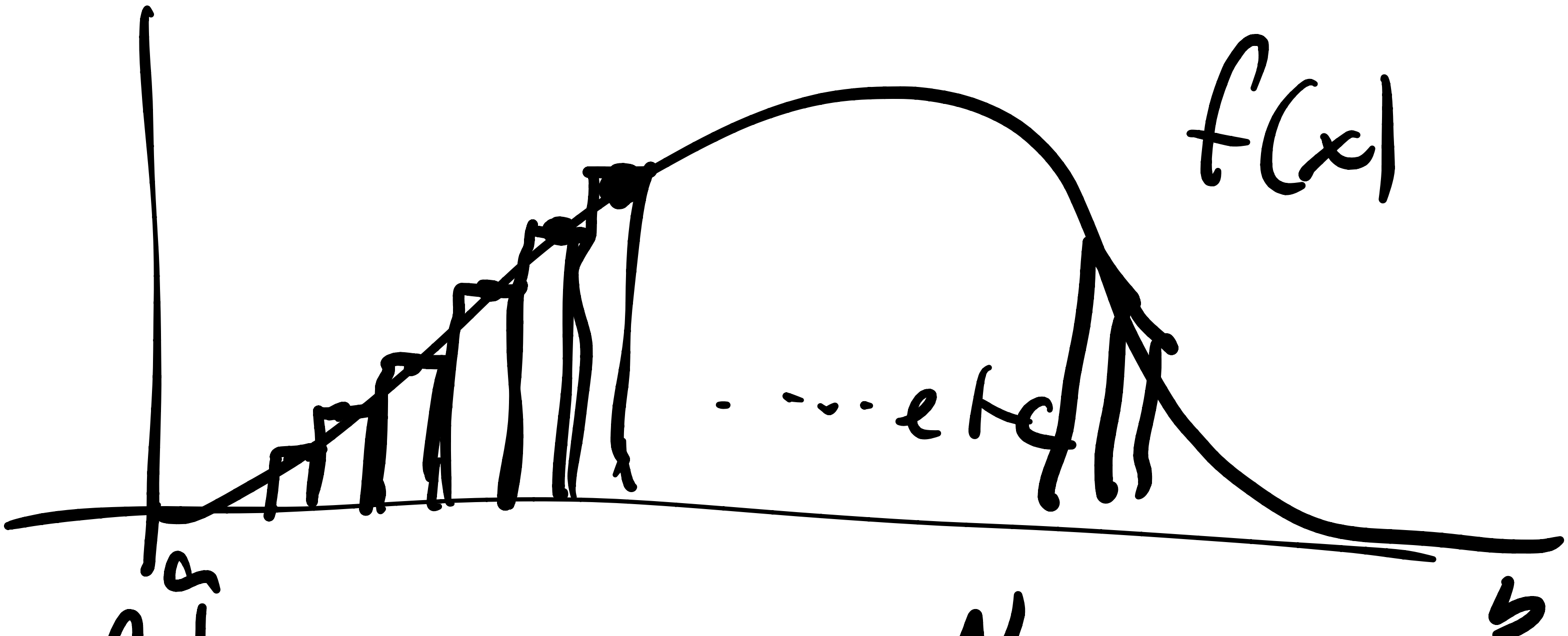
(4) Integrals - anti derivative

$$\int_a^b \frac{df}{dx} = f(b) - f(a)$$

$$\int \frac{df}{dx} = f(x) + C$$

"family of functions"

(5) Integrals are cts version of a sum



has units

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} f(x_i)\,\Delta x$$

if evenly spaced $\quad \frac{1}{N}\cdot(b-a)\sum_{i=1}^{N} f(x_i)$

$$[\text{velocity}] = \frac{[\text{length}]}{[\text{time}]}$$

$$[\text{position}] = \int v\,dt \rightarrow [\text{length}]$$

$$[\text{acceleration}] = \frac{d\,[\text{velocity}]}{dt} \rightarrow \frac{[\text{length}]}{[\text{time}]^2}$$

Chain rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$$

Example $f(x) = x^2 + 2$ $\quad f' = 2x$

$g(x) = (x-1)$ $\quad g' = 1$

$$\frac{d\, f(g(x))}{dx} = f'(g) \cdot g'(x)$$

$$= 2(x-1) \cdot 1$$

$$h = f(g(x)) = (x-1)^2 + 2 = x^2 - 2x + 1 + 2$$

$$h' = 2x - 2 + 0$$

Ex 2: $f(x) = \frac{1}{x} = x^{-1}$

$$f' = -1x^{-2} = -\frac{1}{x^2}$$

$$\frac{d}{dx} \frac{1}{g(x)} = \frac{d}{dx} f(g(x))$$

$$= -\frac{1}{g(x)^2} g'(x)$$

$$\frac{d}{dx} \frac{1}{x^2+1} = -\frac{1}{(x^2+1)^2} \cdot 2x$$

$$\frac{d}{dx}\left(f(x)/g(x) \right) = \frac{-g'(x)f(x) + f'(x)g(x)}{(g(x))^2}$$

product rule

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + g'(x)f(x)$$

Differentials

consider $\frac{df}{dx} \leftarrow \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$\frac{df}{dx} = m$ $\qquad$ $df = m\,dx$

"response" $\qquad$ m – "sensitivity coefficient"

$$df = \left(\frac{df}{dx}\right) dx$$

Chain rule

$$d(fg) = f\,dg + g\,df$$

Integration by parts

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$\int d(fg) = \int f\,dg + \int g\,df$$

$$fg = \int f\,dg + \int g\,df$$

$$\int g\,df = fg - \int f\,dg$$

Partial derivatives

multiple variables

take derivative w/ everything else constant

$$f(x,y) = (x^2+1)y^2$$

$$\left(\frac{\partial f}{\partial x}\right)_y = y^2 \frac{\partial}{\partial x}(x^2+1) = 2xy^2$$

$$\left(\frac{\partial f}{\partial y}\right)_x = (x^2+1)\frac{\partial y^2}{\partial y} = (x^2+1)\,2y$$

← constant

max

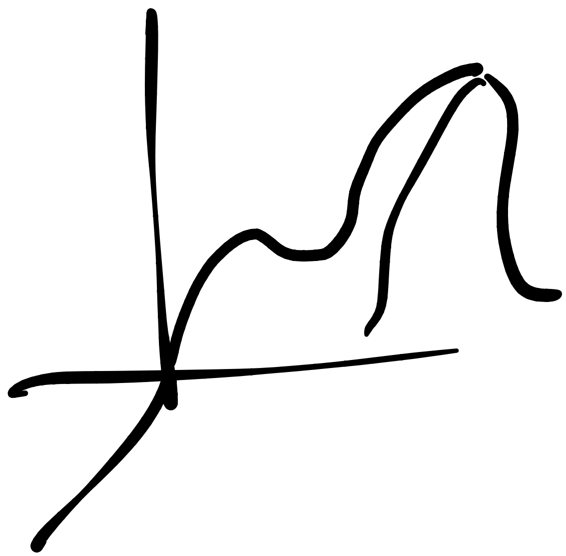
$$\left.\frac{\partial f}{\partial x}\right|_{x^*,y^*} = 0$$

$$\left.\frac{\partial f}{\partial y}\right|_{x^*,y^*} = 0$$

usually, exchange derivative order

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2}{\partial x \partial y} f(x,y)$$

$$d\, f(x,y) = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$$



Consider connection to taylor series....

Intro to thermodynamics

Flow of heat & energy $\leftrightarrow$
how use this to perform work

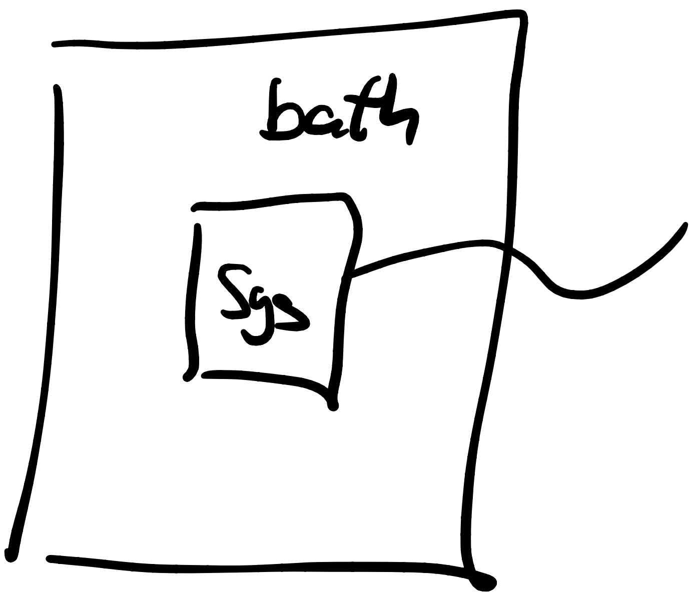
Major concept - "equilibrium"
-> macroscopic state is constant
-> microscopically, all in motion

divide whole universe

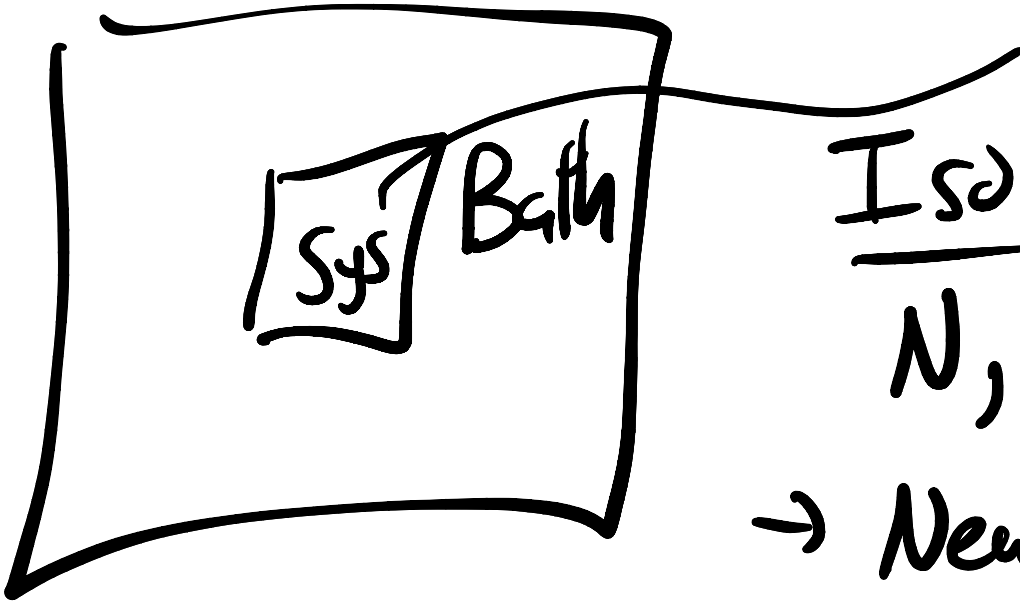
System - what we care about

Environment / bath - everything else

Boundary - imagine
"thermodynamically negligible"



boundary
has properties
isolated
exchange heat
exchange matter



Isolated case

N, V, E ← state

→ Newtons equations

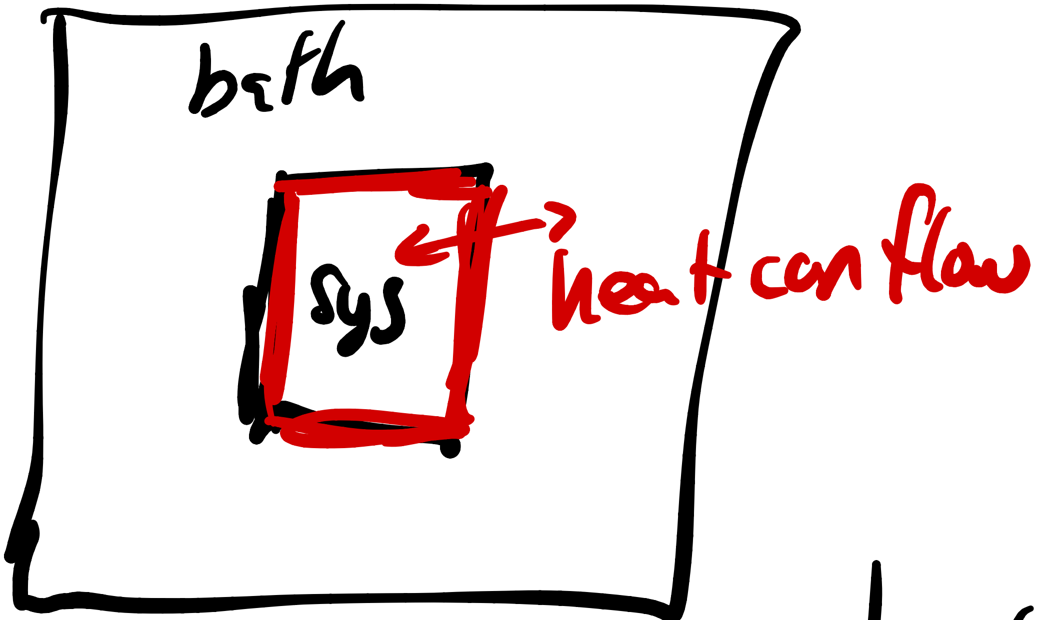
→ Quantum mechanical equations apply

⇒ Energy is constant

Example — state system defined (2 + # components / variables

What can change

Allow energy to go in and out



$dq = 0$ no heat exchange

(adiabatic)

"dia" through

$dq \neq 0$ diathermal

Will see: heat flows until

$$T_{sys} = T_{bath}$$