Lecture 3

Distributions - function chance of an observation in an experiments

Normalized - sum/integral our observations = 1

Last time:

Binomial distribution

chance of "m" successes in "N" trials

something happens with chance p

everything else $1-p$ chance

$$P(n, N) = \frac{N!}{(N-n)!\,n!} p^n (1-p)^{N-n}$$

$\mu = Np$

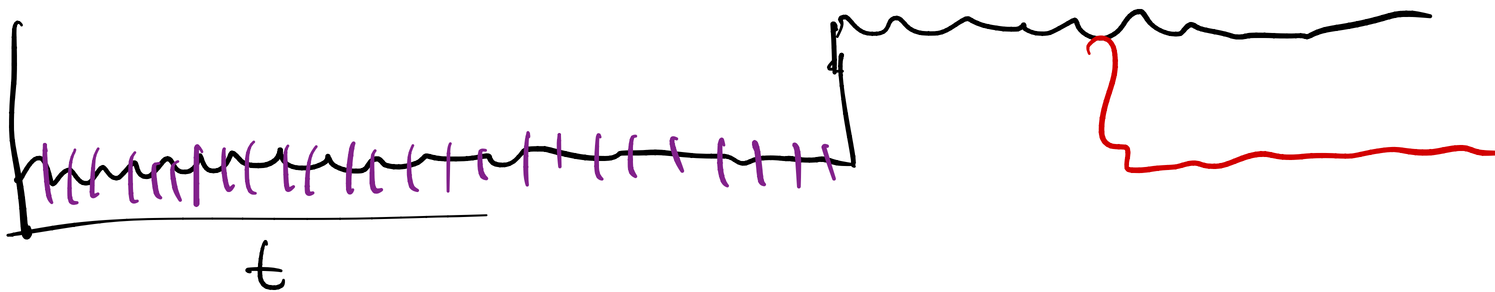
$Var = Np(1-p)$

- Poisson distribution

  limit as $p \to 0$

  and $N \to \infty$

("rare event")



Poisson: λ "rate", number of occurrences on average

μ

$$P(n) = \frac{e^{-\mu}\mu^{n}}{n!}$$ [in some interval]

n times

Example: Radioactive decay

C^{14} – ½ life, 5700 years

Eg: $\underbrace{1\text{ hr}}_{\text{time period}}$,

rate $= \dfrac{\ln(2)}{\tau_{1/2}} \sim 10^{-5}/\text{yr}$

$10^{-5}/\text{yr} \cdot 1\text{ hour}$

hours/yr

$\left[\begin{array}{l} \text{Mean} = \mu \rightarrow \text{unitless} \\ \text{Variance} = \mu \end{array}\right.$

$\mu = $ number of isotopes $\cdot \dfrac{\ln 2}{\text{half life}} \cdot$ time period

aside

$$\frac{d[x]}{dt} = -k[x] \rightarrow [x](t) = [x](0)\, e^{-kt}$$

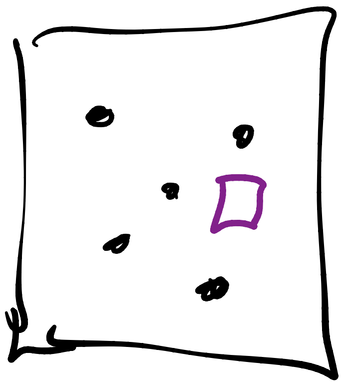
$$x(t)/x(0) = 1/2 \Rightarrow \frac{1}{2} = e^{-kt_{1/2}}$$

$$-\ln 2 = -kt_{1/2}$$

$$t_{1/2} = \ln 2 / k$$

Example 2

space interval



Spacial volume — $(1\,nm)^3$

$$1M = 6\times10^{23}\frac{mc}{l}\times\frac{1l}{1000ml}\frac{1ml}{cm^3}$$

$$= 6\times10^{20}\frac{mc}{cm^3}\times\left(\frac{1cm}{10^7 nm}\right)^3$$

$$= 0.6\,mc/nm^3$$

$1\mu M \sim 6\times10^{-7}\,mc/nm^3$

1M

$$P(1) \sim \frac{e^{-0.6}\, 0.6^{1}}{1!}$$

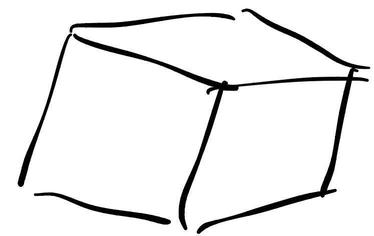
$\mu \sim .6$

$$P(2) \sim e^{-.6} (.6)^2 / 2!$$

$Var \sim .6$

$[H_2O] \sim 55\ M$

33 H_2O mc/nm^3



Sample mean

$$\bar{\mu} = \frac{1}{N_{\text{observations}}} \sum_{i=1}^{N_{\text{observations}}} x_i$$

measurement

$$\sigma^2 = \frac{1}{N_{\text{observations}}} \sum_{i=1}^{N_{\text{obs}}} (x_i - \bar{\mu})^2$$

$\approx$

Notation $\langle X \rangle$ ⇐ average of X

(how?)

$$\sigma^2 = \langle X^2 \rangle - \langle X \rangle^2 = \langle X^2 \rangle - \mu^2$$

For a distribution

moments of distribution

$$\langle X^1 \rangle$$

K possibilities

$$\mu = \sum_{i=1}^{K} X_i P(X_i)$$

$$Var(X) = \langle X^2 \rangle - \mu^2$$

$$\sum_{i=1}^{K} X_i^2 P(X_i)$$

Cts distributions

Gaussian / Normal

$$P(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

$\langle x \rangle = \mu \qquad \langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$

Chance in interval – a to b $\qquad \int_a^b P(x)\,dx$

$$1 = \int_{-\infty}^{\infty} P(x)\,dx$$

moments $\langle x^n \rangle = \int_{-\infty}^{\infty} x^n P(x)\, dx$

Distribution #2
Exponential distribution

$$P(t) = \frac{1}{\tau} e^{-t/\tau}$$

only one parameter

Eg
time until first event exponentially distributed

integral is over all possible values

$$\langle t \rangle = \int_0^\infty t\, P(t)\, dt = \int_0^\infty t \frac{1}{\tau} e^{-t/\tau}\, dt$$

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thermodynamics - little changes

eg dE