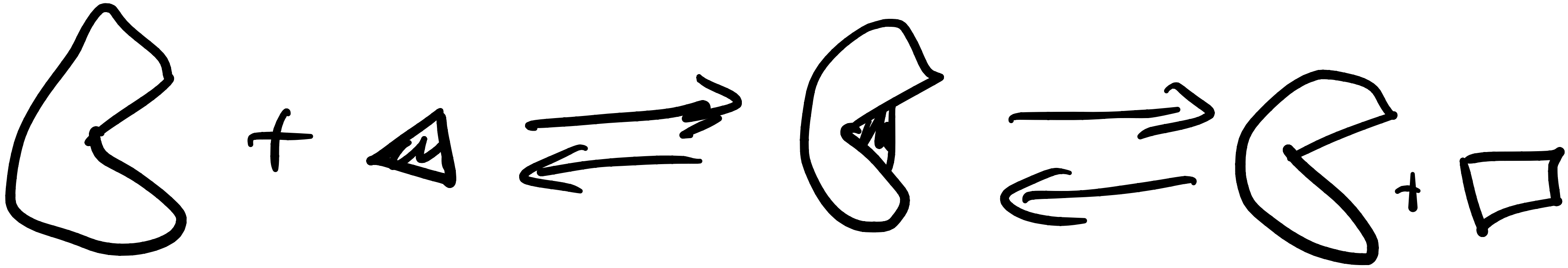
Michaelis-Menten Kinetics



$$E + S \underset{k_b'}{\overset{k_f'}{\rightleftharpoons}} ES \rightleftharpoons E + P$$

$\frac{d[ES]}{dt} = 0$, substrate is in excess

$$[E] + [ES] = [E]_0$$

$$\text{rate of product formation} = \frac{(k_f^1 k_f^2 [S] - k_b^1 k_b^2 [P])[E]_0}{k_f^1 [S] + k_b^1 + k_b^2 [P] + k_f^2}$$

Measure initial rate

Initial conditions: $[P] = 0$, $[S] \approx [S]_0$

$$r_{\text{initial}} = \frac{k_f^1 k_f^2 [E]_0 [S]_0}{k_f^1 [S]_0 + k_b^1 + k_f^2} = \frac{k_f^2 [E]_0 [S]_0}{[S]_0 + K_M}$$

$$K_M = \frac{k_f^2 + k_b^1}{k_f^1}$$

, k_f^2 is called k_{cat}

$$r_{initial} = \frac{k_{cat}[S]_0 [E]_0}{[S]_0 + K_M}$$

If $[S]_0$ really big compared to K_M

$$r_{max} = k_{cat}[E]_0$$

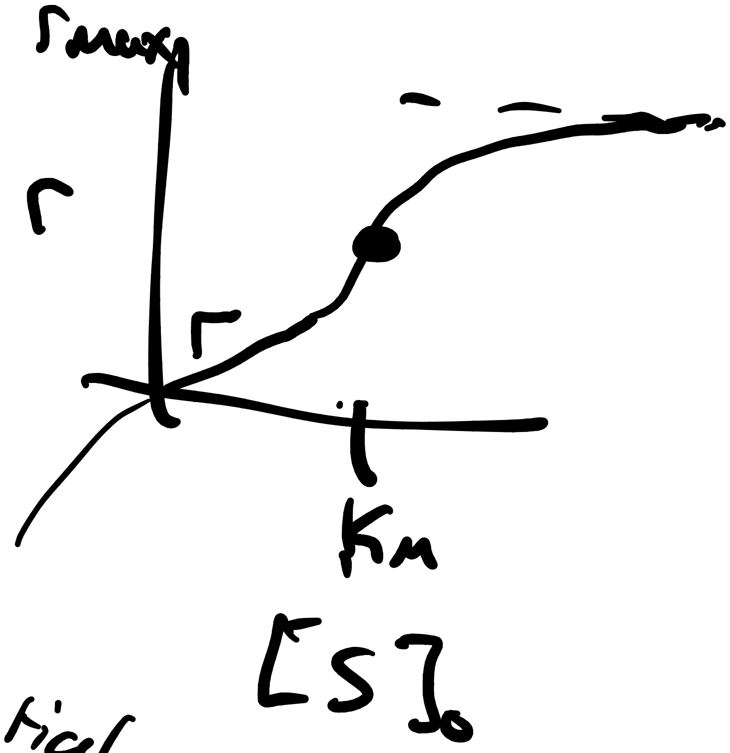
(fit k_{cat})

is $[S_0]$ small, expect rate to be linear in $[S]_0$

$$r = \frac{r_{max} [S]_0}{[S]_0 + K_M}$$

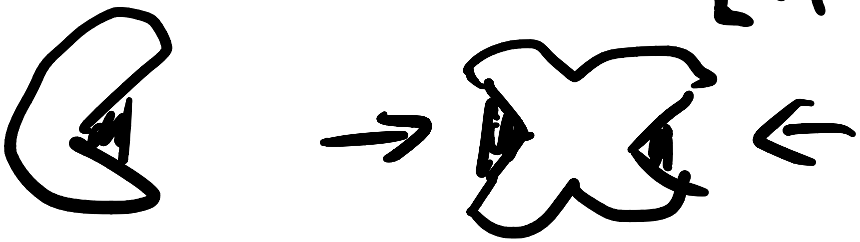
what if $[S]_0 = K_M$,

$$r = r_{max}/2$$



initial slope gives $\frac{r_{max}}{K_M}$

$$\text{Turnover \#} = \frac{r_{max}}{[\text{\# active sites}]}$$



$\uparrow$ $[E]_0 \cdot \#/\text{enzyme}$

Catalytic efficiency

$\epsilon = k_{cat}/K_M$, units of $\frac{1}{M} \cdot \frac{1}{s}$

range $1 - 10^{10} /(Ms)$

$10^8 - 10^{10}$ diffusion limited

$$\frac{k_{cat}}{K_M} = \frac{K_{cat}}{\left(\frac{k_{cat} + k_b'}{k_f'}\right)} = \frac{k_f'}{1 + k_b'/k_{cat}}$$

biggest when k_{cat}, k_f' big + k_b' small

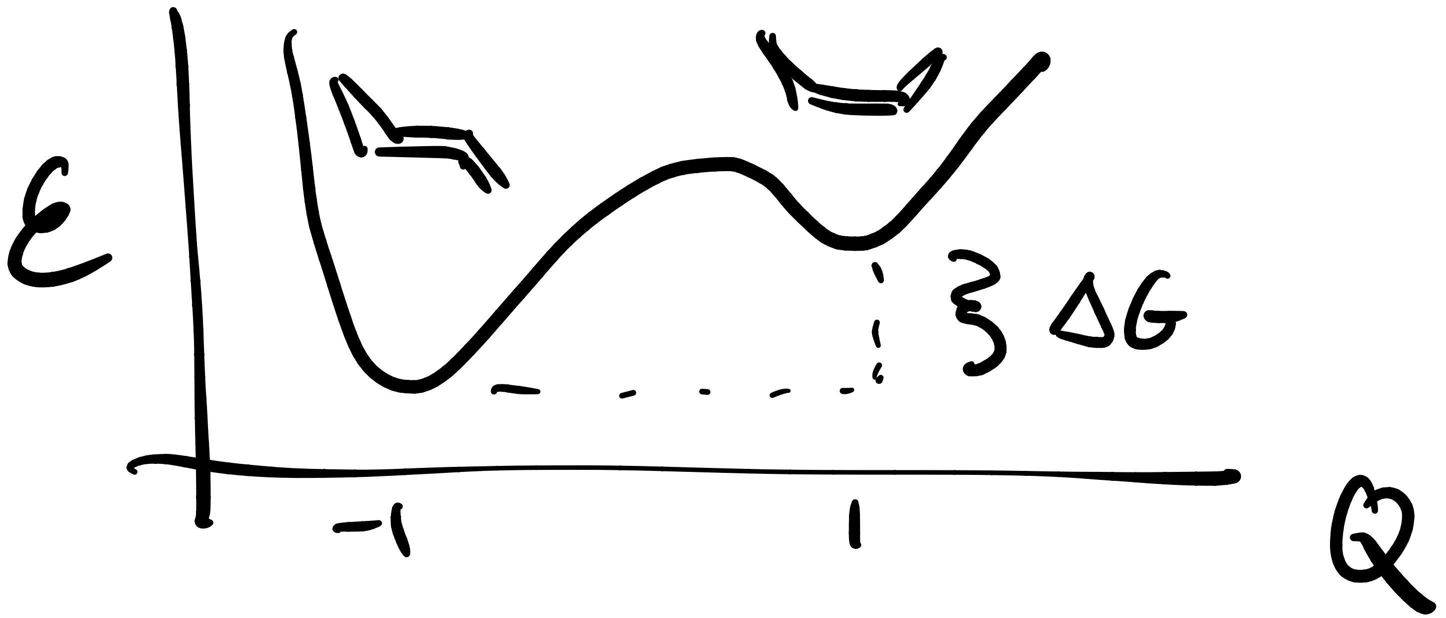
Introduction to Statistical Thermodynamics
(Ch 9)

microscopic → Macroscopic observables

Key idea: Properties are universal
can't depend on arrangement
of molecules

All thermodynamic properties are avgs

cyclohexane

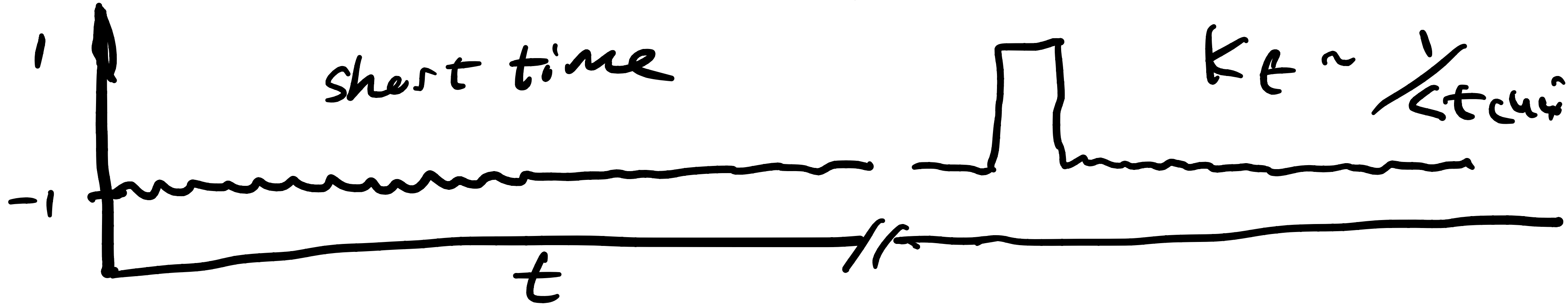


$$\frac{k_f}{k_b} = K_{eq} = e^{-\Delta G^\circ/RT}$$

$$= \frac{[\text{Boat}]}{[\text{Chair}]}$$

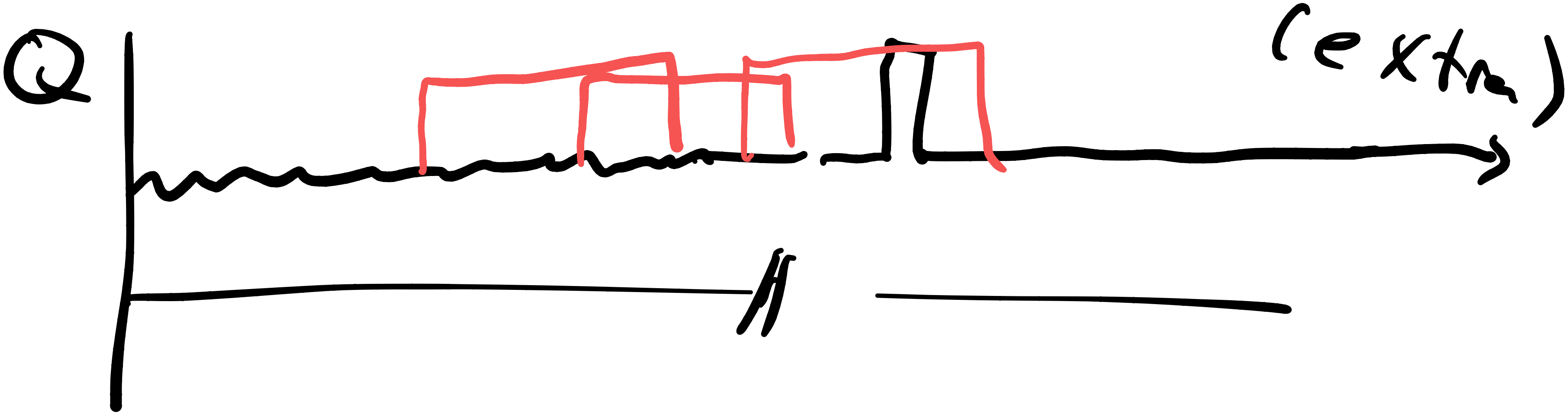
$k_b \sim \frac{1}{\langle t_{boat} \rangle}$

$k_f \sim \frac{1}{\langle t_{chair}}$



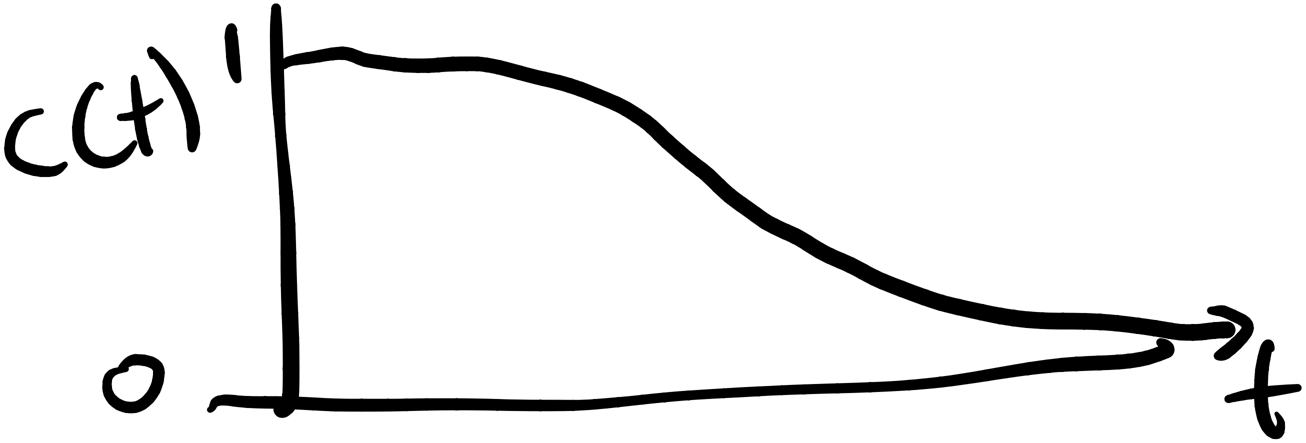
- Single molecule
  observe many transitions → avg
  eq
- Many molecules
  observe few transitions → avg
  eq

time avg = ensemble avg
(assumption/principle)
→ true : ergodic/ergodicity



Time correlation function:

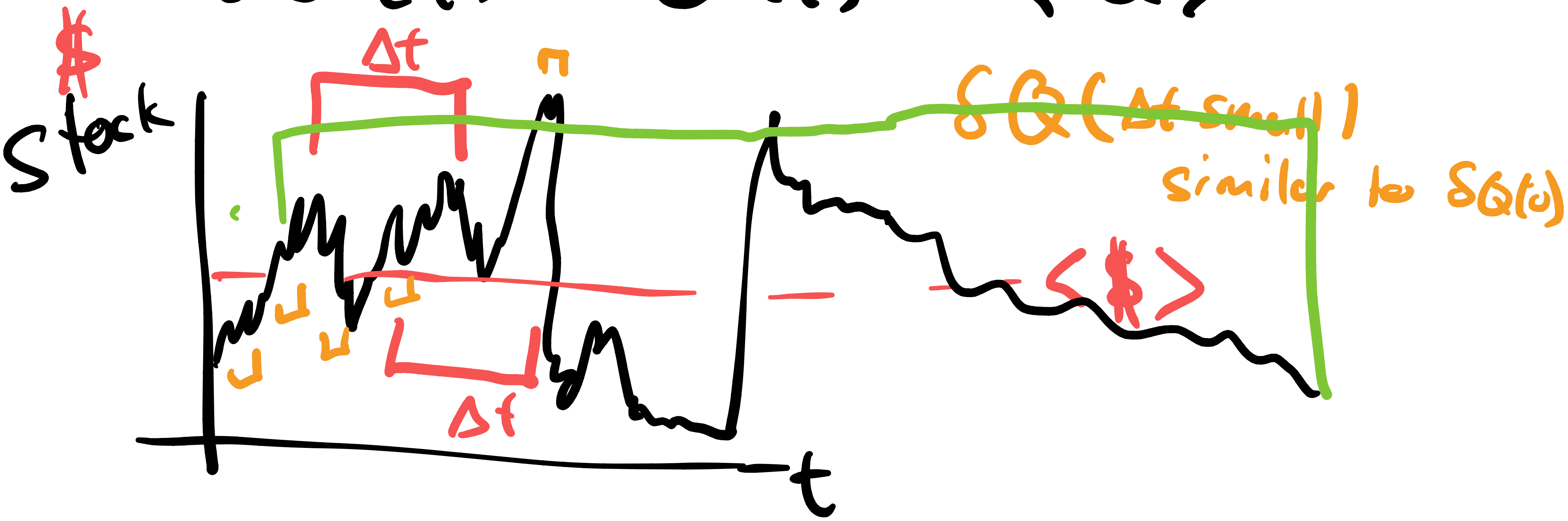
If you know something now,
do you know it later



C(t) is relative to avg

$$C(\Delta t) = \frac{\langle \delta Q(t + \Delta t)\, \delta Q(t) \rangle}{\langle \delta Q(t) \rangle^2}$$

$$\delta Q(t) = Q(t) - \langle Q \rangle$$



$$C(\Delta t) = \frac{\langle \delta Q(t+\Delta t)\, \delta Q(t) \rangle}{\langle \delta Q(t) \rangle^2} \leftarrow \text{variance}$$

$$\langle \delta Q \rangle = 0 = \langle Q - \langle Q \rangle \rangle$$

$$= \langle Q \rangle - \langle \langle Q \rangle \rangle$$

$$= \langle Q \rangle - \langle Q \rangle = 0$$

@ $\Delta t = 0$ $\quad C(t) = 1$

@ Δt really big, no correlation

$$\langle \delta Q(t+\Delta t)\, \delta Q(t) \rangle = \langle \delta Q \rangle \langle \delta Q \rangle = 0$$

what is the avg time for rxn

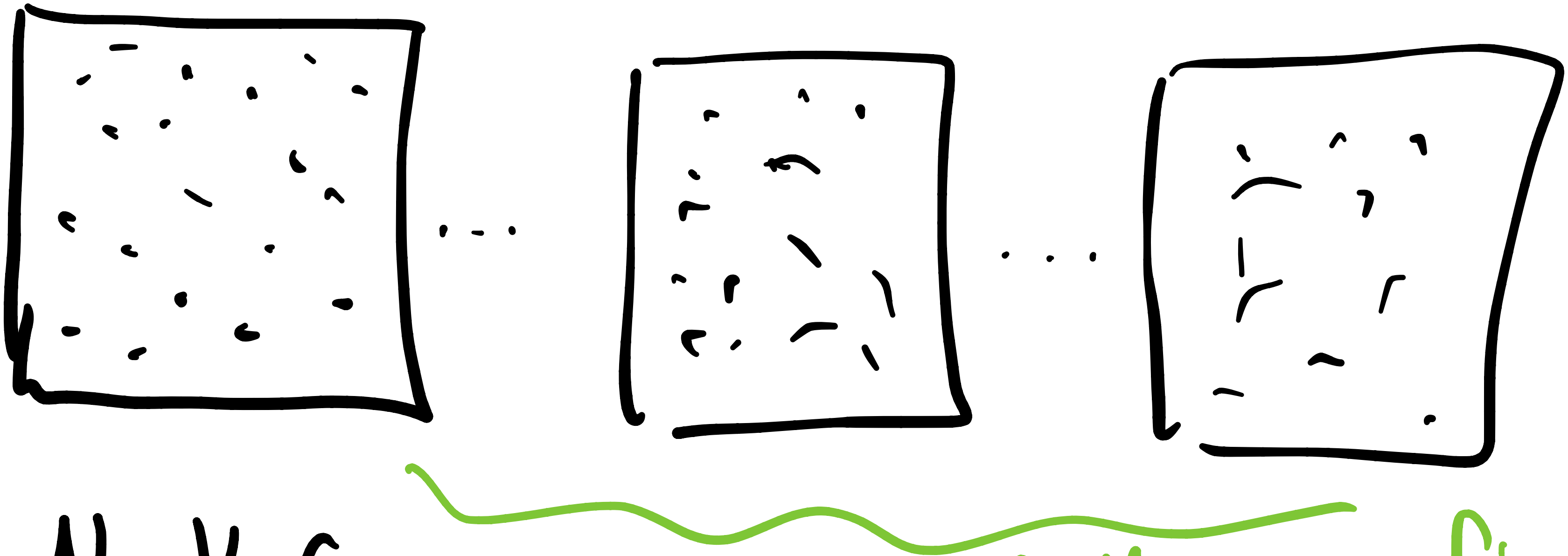
$$\tau_{rxn} \equiv \int_0^\infty C(\Delta t)\, d\Delta t$$

if exponential, $C(\Delta t) = e^{-t/\tau_{rxn}}$

rate $= 1/\tau_{rxn}$

Statistical Ensembles

M copies



N, V, E

all of these configs are equally likely

↖ $P(x_i) = \frac{1}{M}$

Calculate averages over

$$\langle O(x_i)\rangle = \frac{1}{M}\sum_{i=1}^{M} O(x_i)$$

$$\langle O \rangle = \sum_{i=1}^{m} O_i P_i$$ count states

$$\left(= \int dx_1 dx_2 dx_3 \ldots dx_N \, O(\bar{X}) P(\bar{X}) \right)$$

next time, if const T

$$P(X) = e^{-U(x)/k_B T}$$

and similar for other "ensembles"