Adiabatic Expansion

(Skip ~ pg 121)

(Office hour friday schedule?

$đq = 0$, no heat in or out

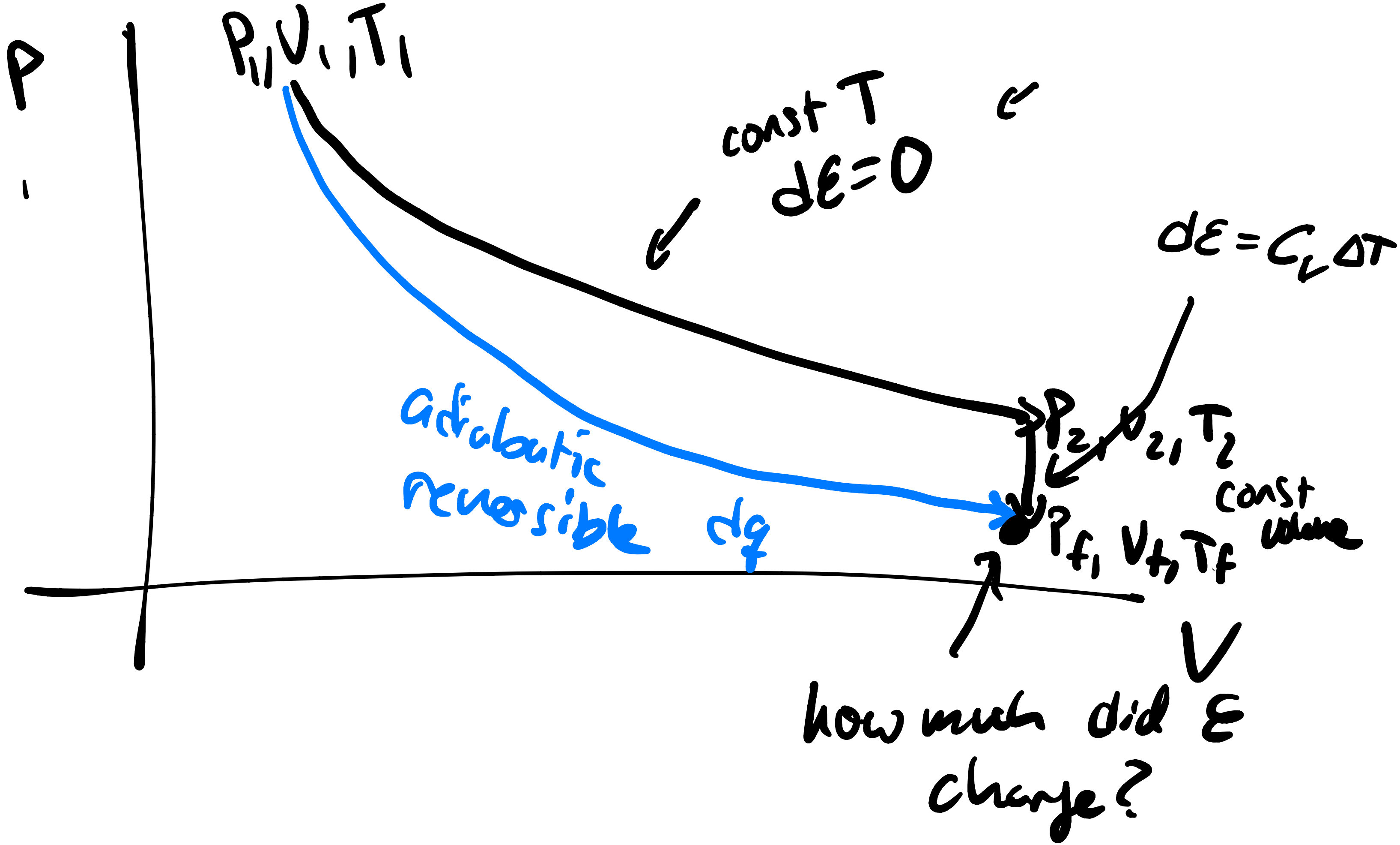
$$dE = \cancel{đq} + đw$$

for an ideal gas $PV = nRT$

$$= -P\,dV$$

$$= -\frac{nRT(V)}{V}\,dV$$

$V\uparrow$ $T\downarrow$ VT


P
V
P_1, V_1, T_1
const T
$dE = 0$
$dE = C_v \Delta T$
P_2, V_2, T_2
const volume
P_f, V_f, T_f
adiabatic
reversible
dq
how much did E
change?

from 2 paths

$-\ln(x) = \ln\left(\frac{1}{x}\right)$

$$\underbrace{-\frac{nRT}{V}\,dV}_{dE \text{ adiabatic}} = C_v\,dT$$

↖ dE for const T then const V

$$\int_{T_i}^{T_f} \frac{C_v}{T}\,dT = -\int_{V_i}^{V_f} \frac{nR}{V}\,dV \Rightarrow$$

$$C_v \ln\left(T_f/T_i\right) = -nR\ln\left(\frac{V_f}{V_i}\right)$$

$$e^{(\ln(x))} = e^{(\ln y)} \Rightarrow x = y$$

$$T_f / T_i = \left(V_i / V_f\right)^{nR/C_V}$$

adiabatic expansion

$$T_f = T_i \underbrace{\left(V_i / V_f\right)}_{<1 \text{ for expansion}}^{nR/C_V}$$

non negative number

$$\frac{P_f V_f}{nR} = T_f \qquad \frac{P_i V_i}{nR} = T_i$$

$$\frac{P_f V_f}{P_i V_i} = \frac{T_f}{T_i} = \left({}^{V_i}/_{V_f} \right)^{nR/C_v}$$

$$P_f / P_i = \frac{V_i}{V_f} \left({}^{V_i}/_{V_f} \right)^{nR/C_v} = \left({}^{V_i}/_{V_f} \right)^{\left[\frac{nR}{C_v} + 1\right]}$$

$$P_f / P_i = \left(V_i / V_f \right)^{\frac{nR}{C_V} + 1}$$

$$\Rightarrow P_f = P_i \left(\frac{V_i}{V_f} \right)^{\gamma > 1}$$

$$\frac{V_i}{V_f} < 1$$

$$\frac{nR}{C_V} + 1 = \frac{nR + C_V}{C_V} = \frac{C_P}{C_V} = \gamma$$

for monatomic ideal gas $\frac{5/2}{3/2} = 5/3$

General formula for adiabatic expansion/compression

$$P(V) = P_i V_i^{\gamma} / V^{\gamma}$$

[const T
$P \propto 1/V$

$$= \lambda / V^{\gamma} = \lambda V^{-\gamma}$$

$\lambda \leftarrow$ const

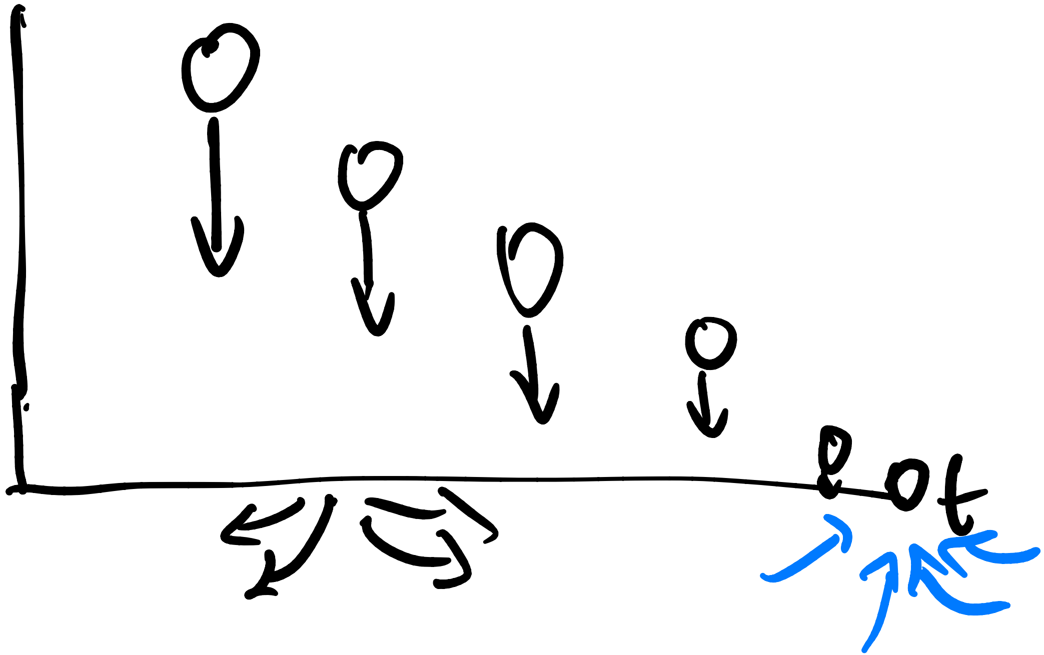
$$W = -\int_{V_i}^{V_f} P(V)\,dV = \frac{P_i V_i^{\gamma}}{\gamma - 1}\left(\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right)$$

Second law of thermodynamic & entropy

Total energy is conserved

Spontaneous process

What sets direction of spontaneous processes?



Sound / friction

Energy spreads out → creation of entropy

Original law (Claussius)

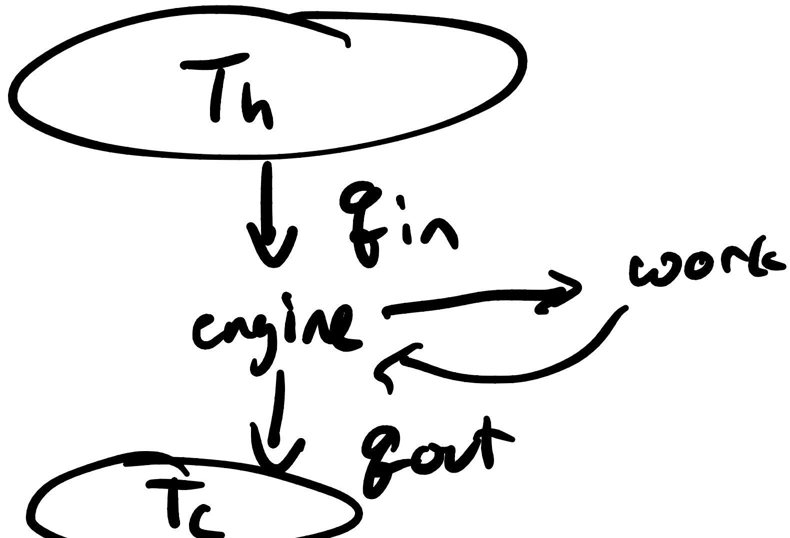
no process is possible where heat transfers from cold object to a hot object

Engine a "cycle" (ends where it starts)

which converts Energy from one form to another (Mechanical work)

Diagram

simple engine



Define "efficiency"

$$\epsilon = \frac{W_{done}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

Can we analyze ϵ for some "ideal" engine

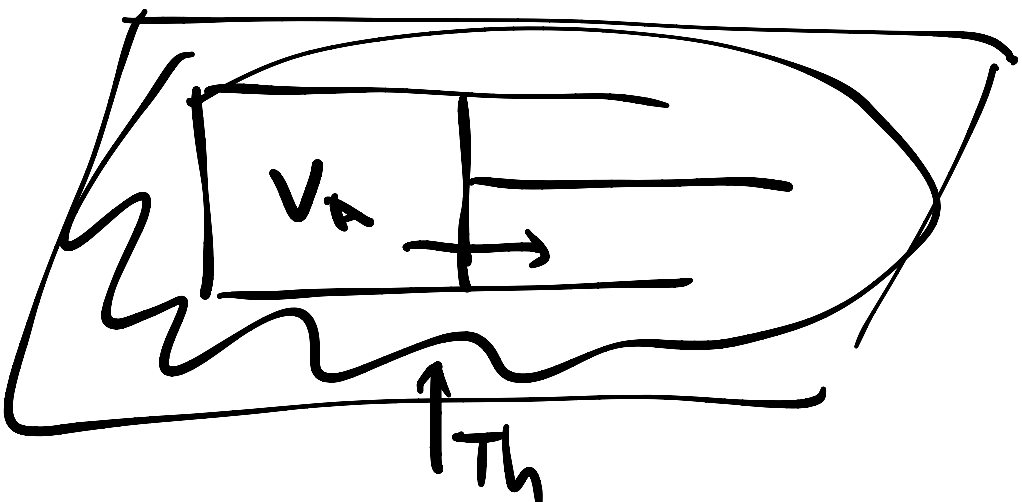
"Carnot" Engine

[otto cycle see practice midterm]

Carnot engine

4 steps in a cycle, all reversible

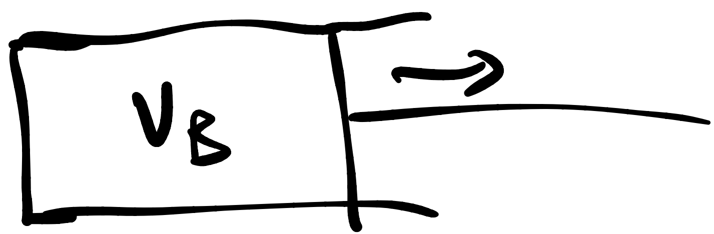
a) isothermal expansion at T_h



q in

work out

Step 2 adiabatic expansion



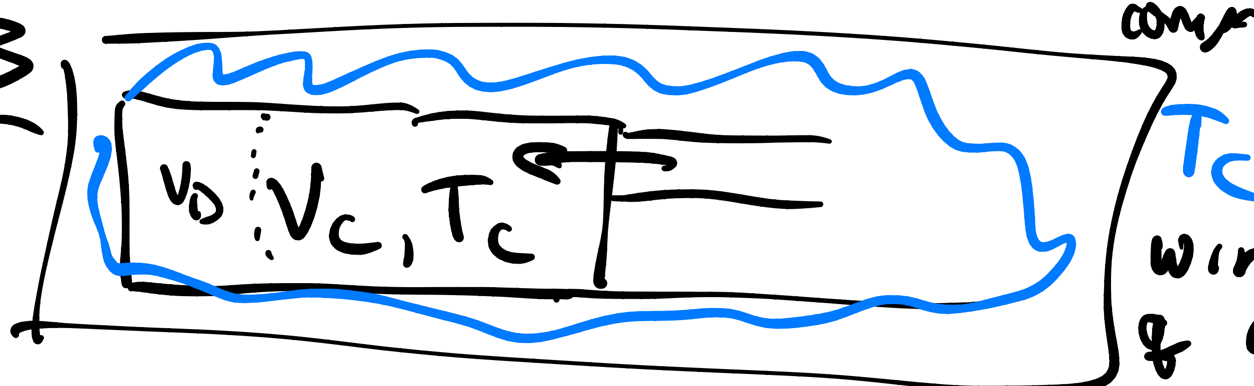
$q = 0$

w out

cooling off until @ T_c

const T compression

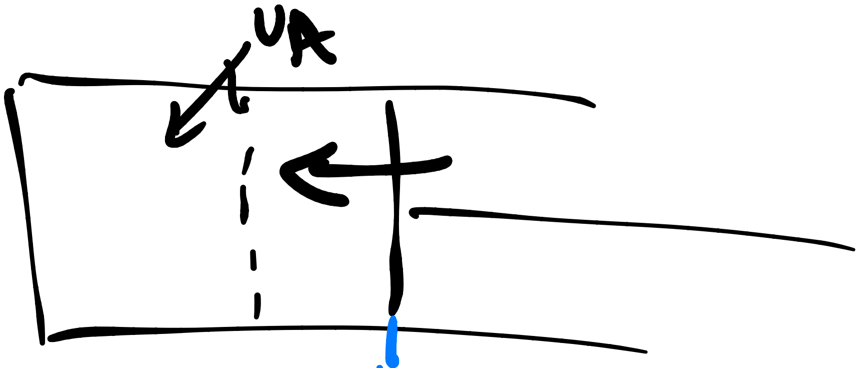
Step 3



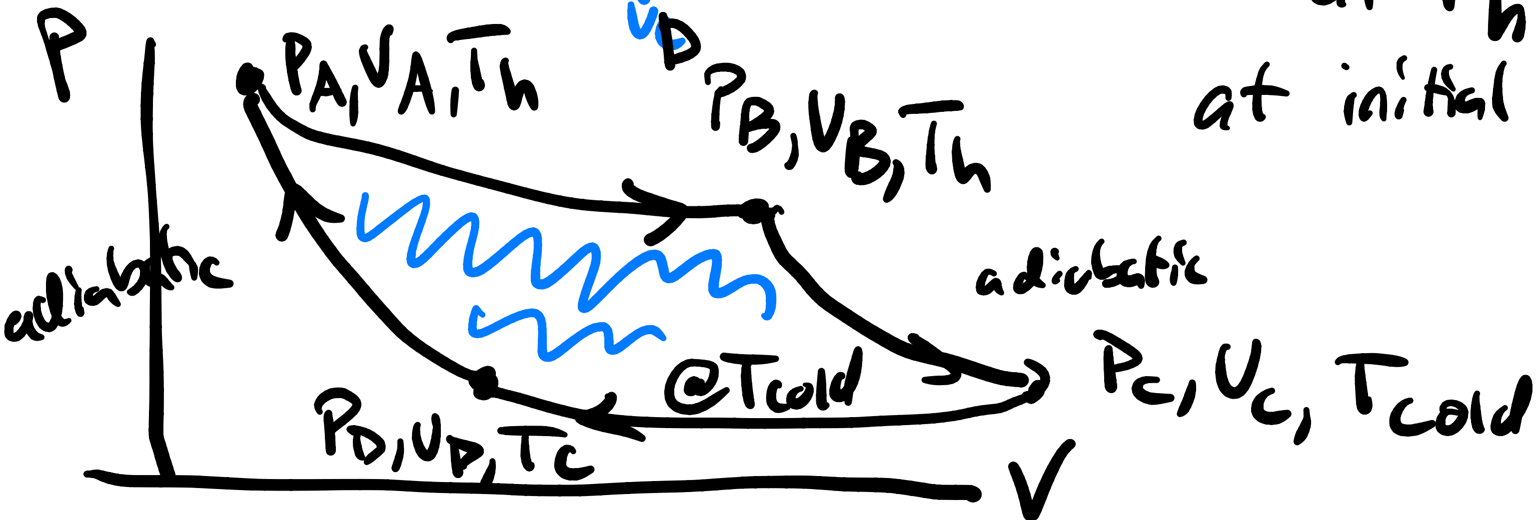
w in

q out

Step 4 adiabatic compression



T goes up until back at T_h at initial volume



	q	w_{sys}
① isothermal expansion	$nRT_h \ln(V_B/V_A)$	$-nRT_h \ln(V_B/V_A)$
② adiabatic expansion	0	$\Delta E = C_v(T_c - T_h)$
③ isothermal compression	$nRT_c \ln(V_D/V_C)$	$-nRT_c \ln(V_D/V_C)$
④ adiabatic compression	0	$C_v(T_h - T_c)$

ideal gas
$dE = 0,\ q = -w$

$$W_{done} = -W_{sys} = nRT_h \ln(V_B/V_A) + nRT_c \ln(V_D/V_C)$$

$$= q_1 + q_3$$

$$\epsilon = \frac{W}{q_{in}} = \frac{q_1 + q_3}{q_1} = 1 + q_3/q_1$$

$$q_3/q_1 \overset{IG}{=} \frac{nRT_c \ln(V_D/V_C)}{nRT_h \ln(V_B/V_A)}$$

$$\left(q_3/q_1\right)^{IG} = \frac{\cancel{nR}\,T_c \ln(V_D/V_C)}{\cancel{nR}\,T_h \ln(V_B/V_A)}$$

for adiabatic expansions

$$V_f/V_i = (T_f/T_i)^{-C_V/nR}$$

$$\Rightarrow \ln(V_D/V_C)/\ln(V_B/V_A) = -1$$

$$\boxed{\epsilon_{carnot} = 1 - T_c/T_h}$$

beyond IG