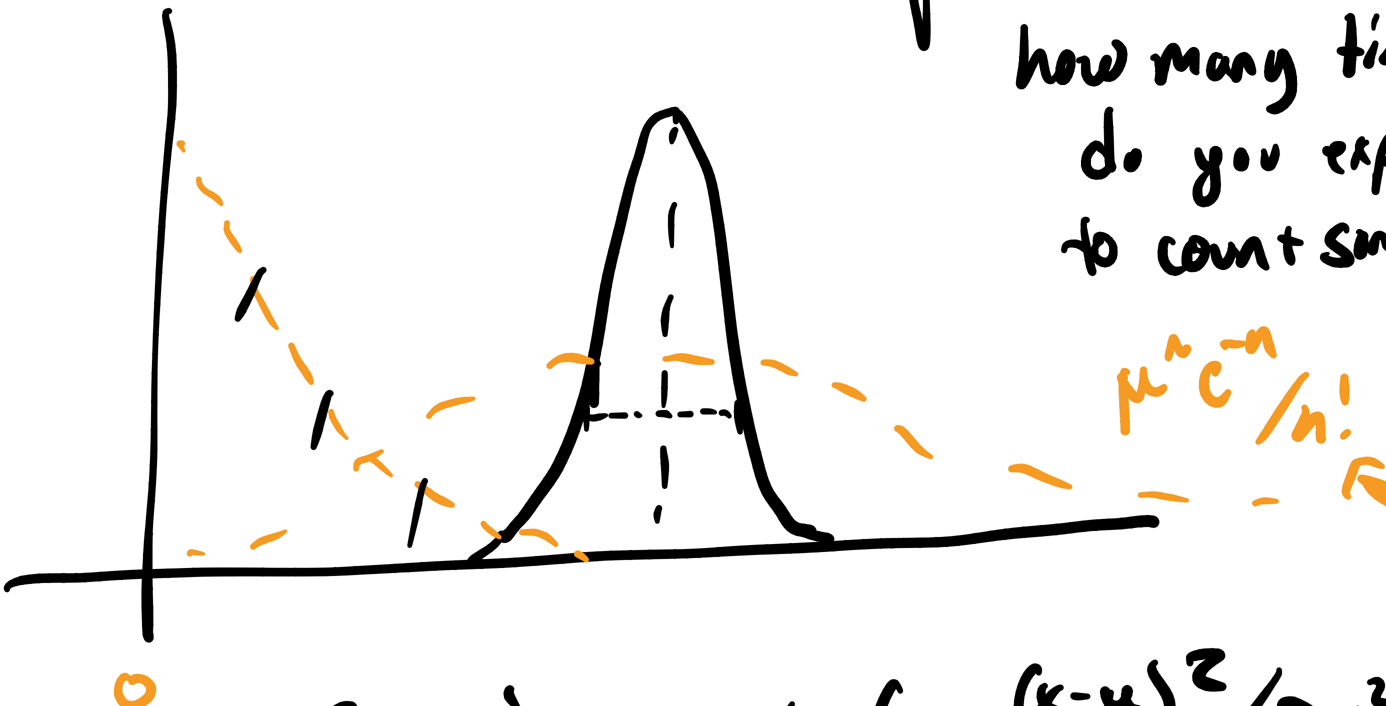
Distributions recap

poisson:
how many times do you expect to count something



$\mu^n e^{-n} / n!$

Gaussian $\exp\left(-(x-\mu)^2 / 2\sigma^2\right)$
sum of random numbers

$$P(n, N) = \binom{N}{n} p^n (1-p)^{N-n}$$

Derivative rules

$f(x)$ $\qquad$ $g(x)$

$$\frac{d}{dx} f(g(x)) = g'(x) \cdot f'(g(x))$$

$g(x) = -x^2$

$$\frac{d}{dx}\left[e^{-x^2}\right] = \frac{d}{dx}\left[e^{g(x)}\right] = e^{g(x)} g'(x)$$

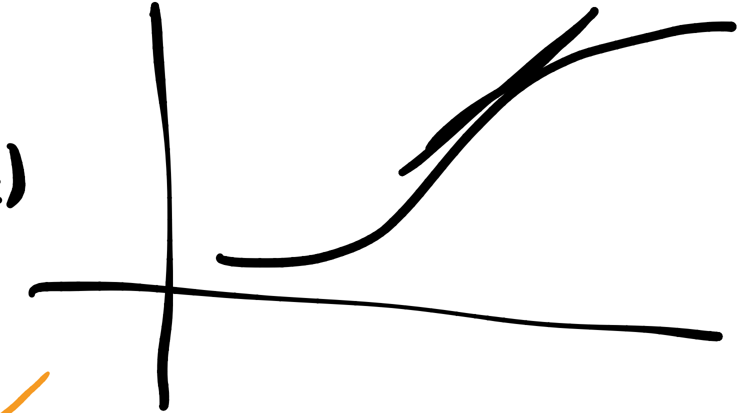
$$= -2x e^{-x^2}$$

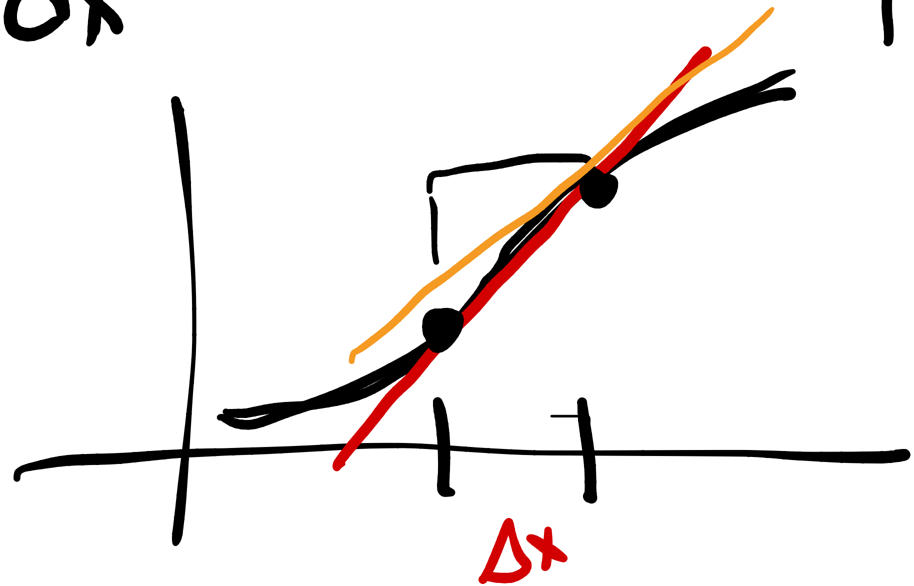
$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f(x)\,g'(x) + g(x)\,f'(x)$$

$$\frac{d}{dx}\left[x^2 e^{-x^2}\right] = x^2\left(-2xe^{-x^2}\right) + 2xe^{-x^2}$$

Differentials

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



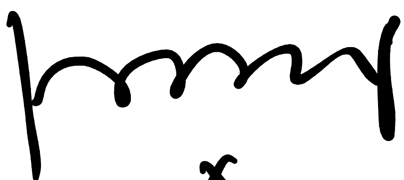


$$\frac{df}{dx} = 5$$

$$df = 5\,dx$$

$$\frac{df}{dx} = m \Rightarrow df = m \; dx$$

sensitivity



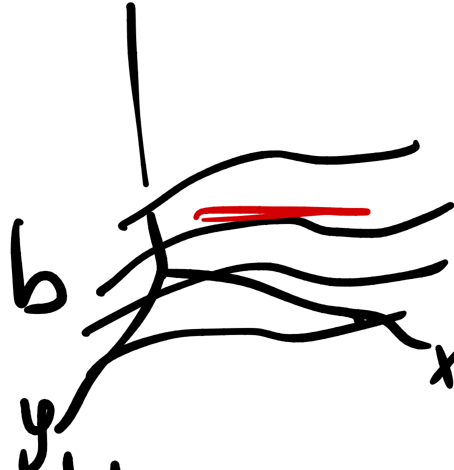
x

$$F(x) = kx$$

$$dx = \frac{1}{k} dF$$

$$\mathcal{E}(a, b)$$

$$d\mathcal{E} = m\, da + n\, db$$



$$= \left(\frac{\partial \mathcal{E}}{\partial a}\right)_b da + \left(\frac{\partial \mathcal{E}}{\partial b}\right)_a db$$

$$f(x, y)$$

$$\frac{\partial f}{\partial x} f(x, y) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

$$f(x,y) = (x^2+1)\,y^2$$

$$\left(\frac{\partial f}{\partial x}\right)_y = 2x\,y^2 + (x^2+1)\overbrace{\left(\frac{\partial y^2}{\partial x}\right)_y}^{=0}$$

$$\left(\frac{\partial f}{\partial y}\right)_x = 2y\,(x^2+1)$$

$$\left[\frac{\partial A}{\partial B} = \left[\frac{\partial B}{\partial A}\right]^{-1}\right]$$

most cases

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x} f(x,y)\right) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y} f(x,y)\right) \overset{\text{eg}}{=} 4xy$$

differentials are useful notation

$$d(fg) = g\,df + f\,dg$$

$$\underbrace{\int d(fg)}_{fg} = \int g\,df + \int f\,dg$$

$$\int f\,dg = fg - \int g\,df \Rightarrow \int u\,dv = uv - \int v\,du$$

Taylor Series

$$f(x+dx) = f(x) + \frac{\partial f}{\partial x} dx$$

$$+ \frac{1}{2} \frac{\partial^2 f}{\partial x^2} (dx)^2 + \frac{1}{3!} \frac{\partial^3 f}{\partial x^3} (dx)^3 + \dots$$

$$e^{(dx)} = \underbrace{1 + (dx) + \frac{(dx)^2}{2}}_{} + \frac{(dx)^3}{3!} + \dots$$

close to $x=0$

$x=0$

Intro to thermodynamics Ch 3

Thermodynamics - motion of heat

wanted to produce work

Developed way before atoms established

Major Concept Equilibrium

"State" of system is constant

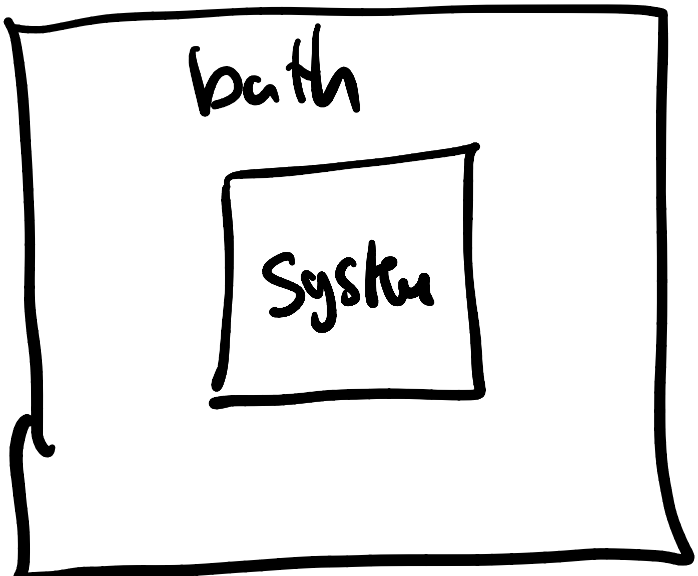
Key idea divide universe

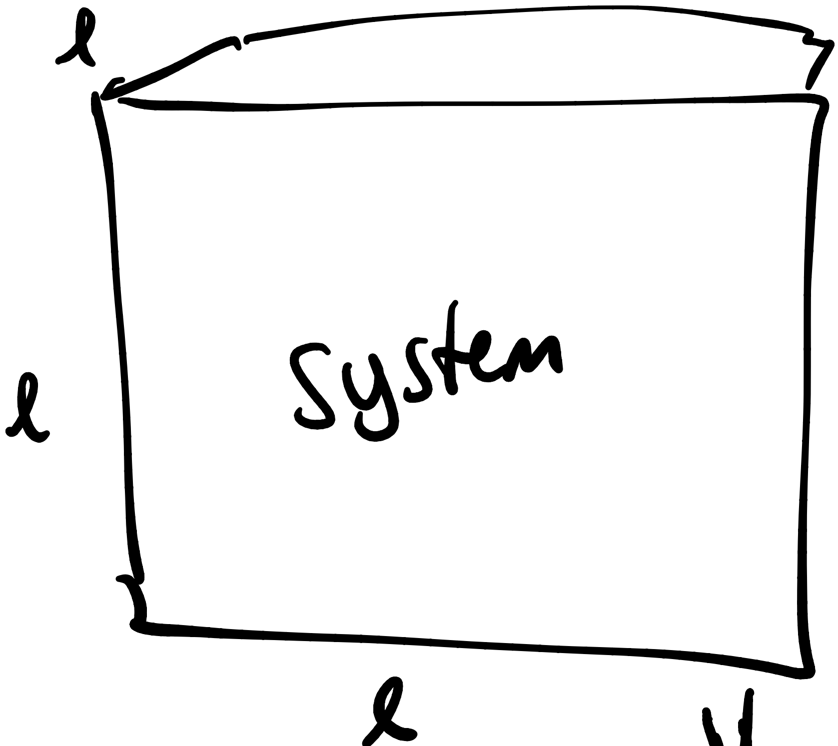
system ← want to study

everything else "bath"

(boundary) – "thermodynamically negligible"

System - bath coupling via boundary





Isolated

N molecules inside

$(N_A, N_B, \ldots)$

fixed Volume

$V = l^3$

"isolated" not connected to any bath

consequence:

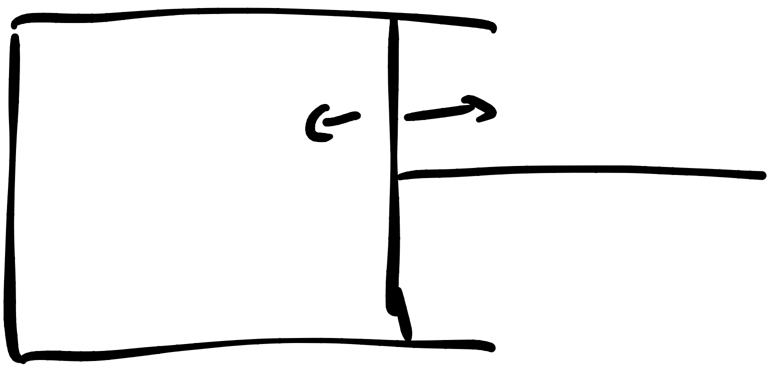
molecules follow newton's equation

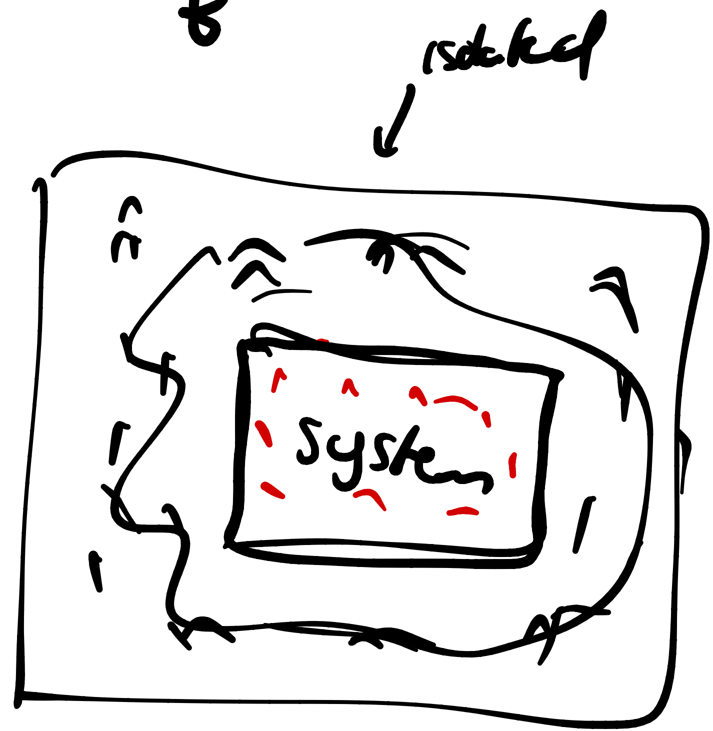
$\Rightarrow$ energy is constant

N, V, E

what can we change? "q" heat

volume



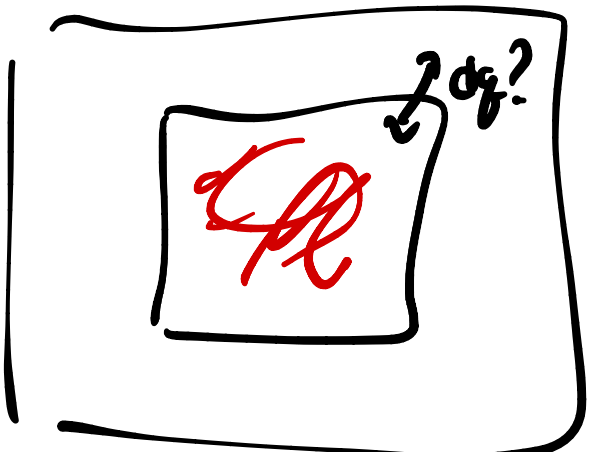


total N, V, E fixed
but energy can flow

Allowed changes

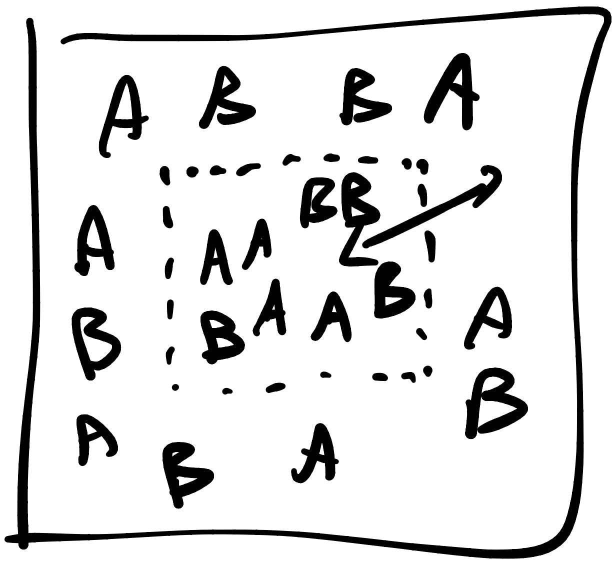
$dq = 0$ adiabatic – no heat passes through

$dq \neq 0$ diathermal – heat passes



is system "insulated"

Eq. is when $T_{inside} = T_{outside}$



$dN = 0$ closed

$dN \neq 0$ open

if open,

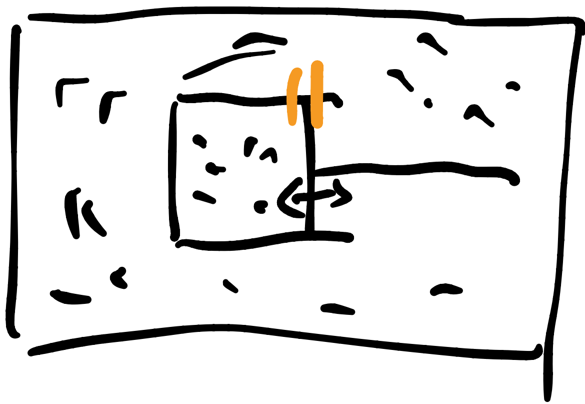
diathermal

number of molecules changes

until μ ← chemical potential

$\mu_{in} = \mu_{out}$

Volume can change



$dU \neq 0$ isobaric

same pressure

$dN = 0$

$dq = 0$

$dV = 0$ isochoric

same space

Equilibrium piston moves until $P_{inside} = P_{outside}$

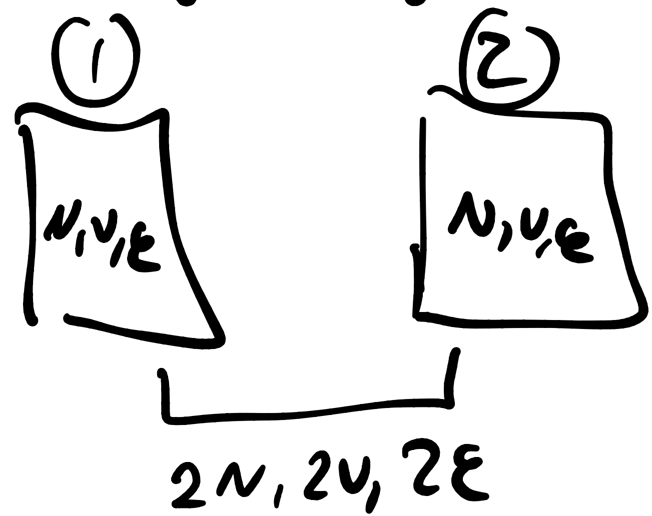
E, V, N ← extensive

[S] C

$T \quad P \quad \mu \quad 1/k$

e v n s c

copy of system



intensive

don't change w/ system size

N/V ← density

E/V ← energy density