Last time

sequence: H T T H . . . . T T

prob N_H in N events

(N tails)

$$\text{Prob} = P_H \cdot P_T \cdot P_T P_H \cdots P_T P_T$$

Independent:

$$= P_H^{N_H} \cdot P_T^{N_T} = P_H^{N_H} (1 - P_H)^{N - N_H}$$

Last time (2)

number of ways to get m things in n trials: n choose m $\binom{n}{m}$

$$\binom{n}{m} = \frac{n!}{m!\,(n-m)!}$$

prob of N_H in N flips $= \binom{N}{N_H} P_H^{N_H} (1-P_H)^{N-N_H}$

"Binomial distribution"

for case of a coin, or r.w.

$p = 1/2$

In that case $P(N_H, N) = \binom{N}{N_H} p^{N_H} (1-p)^{N-N_H}$

$= \binom{N}{N_H} \left(\frac{1}{2}\right)^N$

why called binomial distribution

$(a+b)^N =$

$$(a+b)^2 = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$\begin{aligned}(a+b)^2(a+b) &= a^3 + a^2b + aba + ab^2 \\ &\quad + ba^2 + bab + b^2a + b^3 \\ &= a^3 + 3a^2b + 3ab^2 + b^3\end{aligned}$$

binomial coefficients

pascal's triangle

$$\begin{array}{lccccccccc}
0 & & & & & 1 & & & & \\
1 & & & & 1 & & 1 & & & \\
2 & & & 1 & & 2 & & 1 & & \\
3 & & 1 & & 3 & & 3 & & 1 & \\
4 & 1 & & 4 & & 6 & & 4 & & 1 \quad \cdots
\end{array}$$

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0}\binom{2}{1}\binom{2}{2}$$

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$(a+b)^N = \sum_{m=0}^{N} \binom{N}{m} a^m b^{N-m}$$

Probabilities add up to 1

prob of N_H in N trials

$$P(N_H, N) = \binom{N}{N_H} p^{N_H} (1-p)^{N-N_H}$$

$$1 \overset{?}{=} \sum_{N_H=0}^{N} \binom{N}{N_H} p^{N_H} (1-p)^{N-N_H}$$

1^N

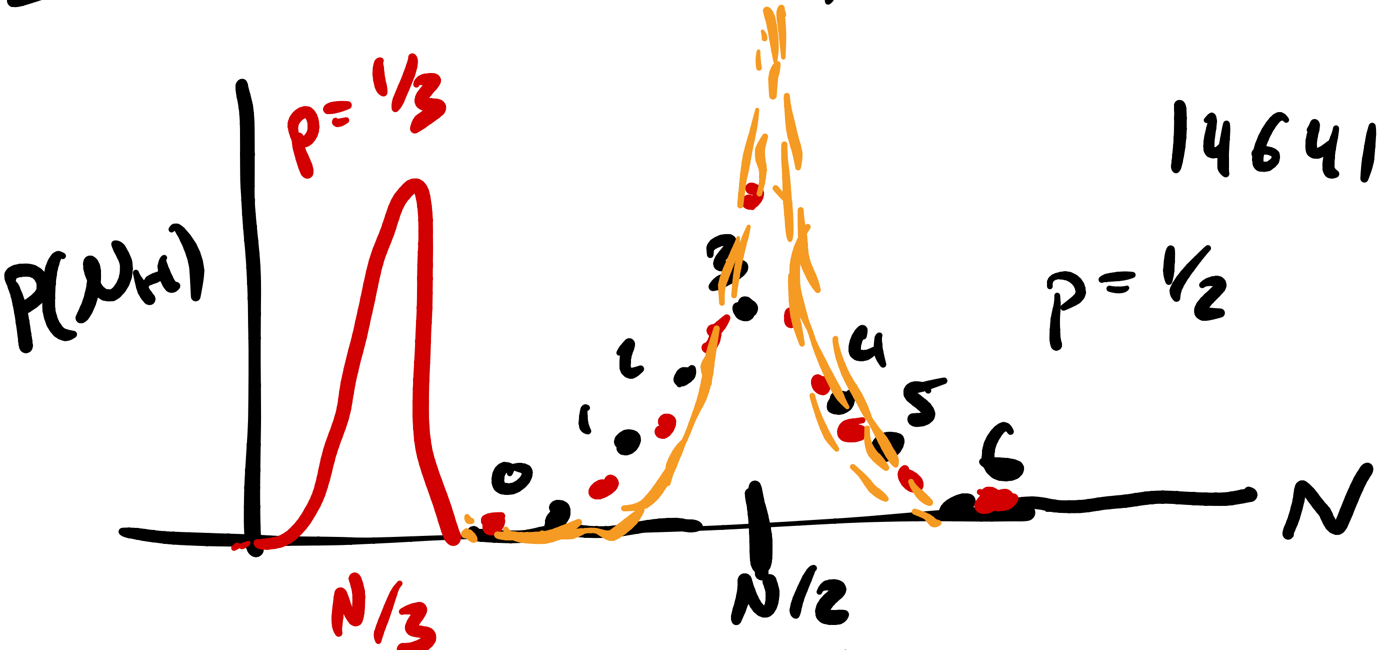
$$(a+b)^N = \sum_{m=0}^{N} \binom{N}{m} a^m b^{N-m}$$

$a = p$

$b = 1-p$

Distribution

$$P(N_H, N) = \binom{N}{N_H} p^{N_H} (1-p)^{N}$$



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$p = 1/2$

average : $p \cdot N$

Average, Variance

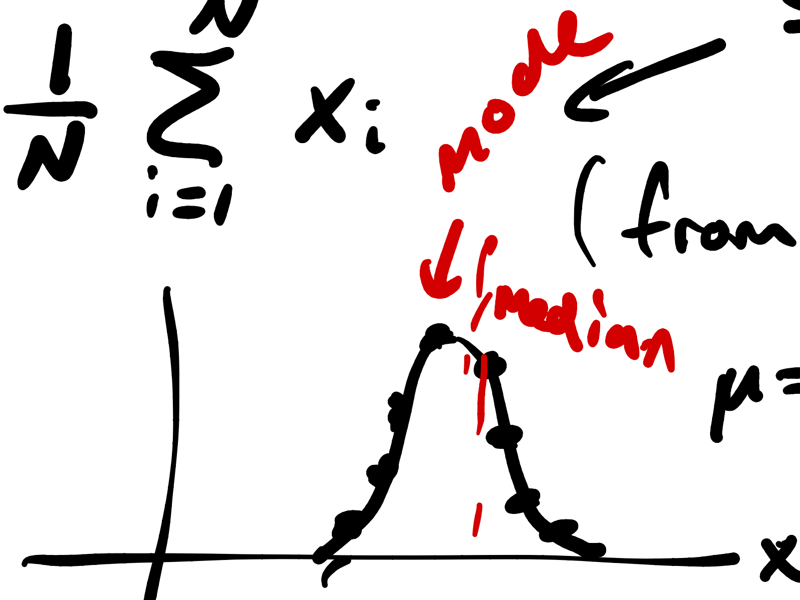
$$\{x_1, x_2, \dots, x_N\}$$

average (mean)

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

sample mean (from data)

$P(X)$



$$\mu = \sum_{n=1}^{N} X_i P(X_i)$$

Variance

data

$$\sigma^2 = \mathrm{Var}(\ ) = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$$

standard deviation

HW?

$$= \mathrm{Avg}[x^2] - (\mathrm{Avg}[x])^2$$

μ^2

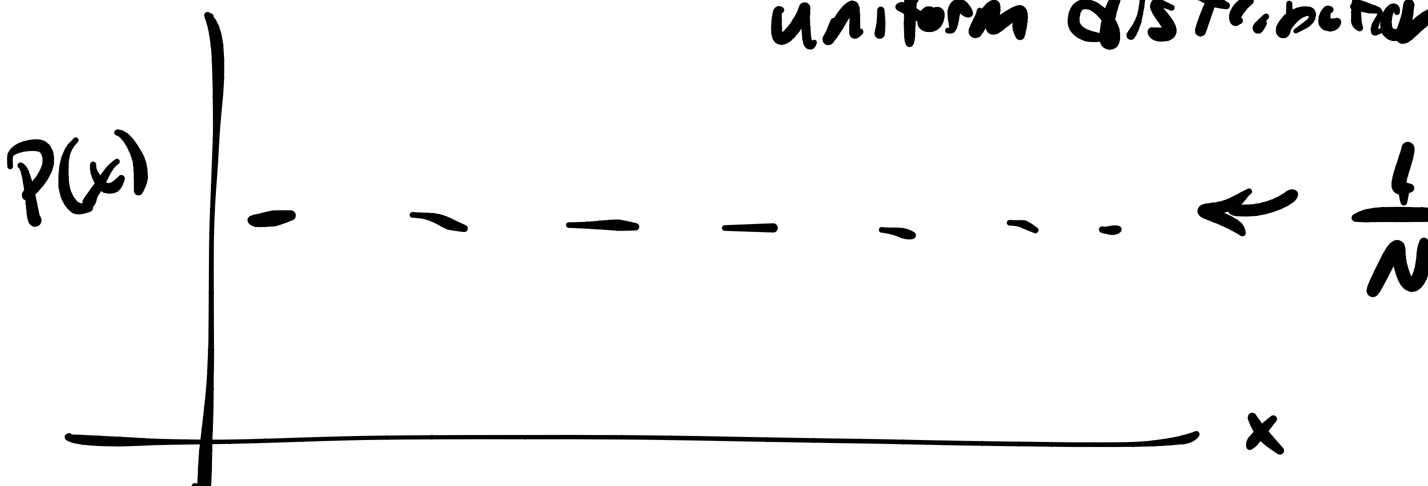
$$\mathrm{Avg}(\ ) = E[\ ] = \underbrace{\langle x \rangle}_{\text{average of } x}$$

expectation

for a distribution

$$\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2 P(x_i)$$

uniform distribution



Probability distribution function P

① $1 \geq P(x_i) \geq 0$

② nomalized

$$\sum_i P(x_i) = 1$$

eg Uniform

$$P(x_i) = \frac{1}{N}$$

$$\sum_{i=1}^{N} \frac{1}{N} = 1$$

poisson

$$P(n, N) = \binom{N}{n} p^n (1-p)^{N-n}$$

poisson

$$P(n,N) = \binom{N}{n} p^n (1-p)^{N-n}$$

$$\mu = \sum_{n=0}^{N} n\, P(n) \quad = N \cdot p$$

$$\sigma^2 = \sum_{n=0}^{N} (n-\mu)^2 P(n) = N \cdot p \cdot (1-p)$$

for $p = \frac{1}{2}$, $\sigma^2 = N \cdot \frac{1}{4}$

$$\mu \propto N$$

$$\sigma \propto \sqrt{N}$$

"central limit theorem"

$$\sigma/\mu \sim \frac{\sqrt{N}}{N} \sim \frac{1}{\sqrt{N}}$$

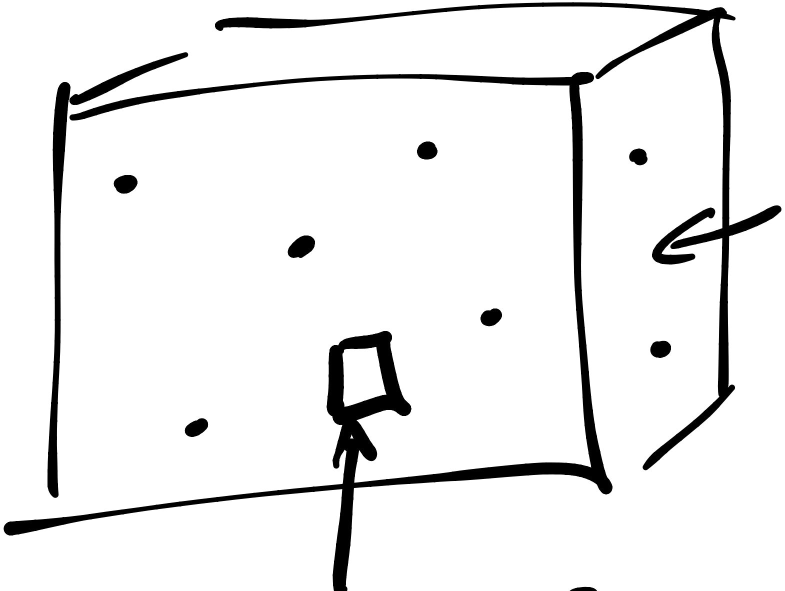
goes to zero with big N

Poisson distribution

prob of n things happening
in a fixed interval
unlikely event "rare event"

$$P(n,\mu) = \mu^n e^{-\mu} / n!$$

μ avg number



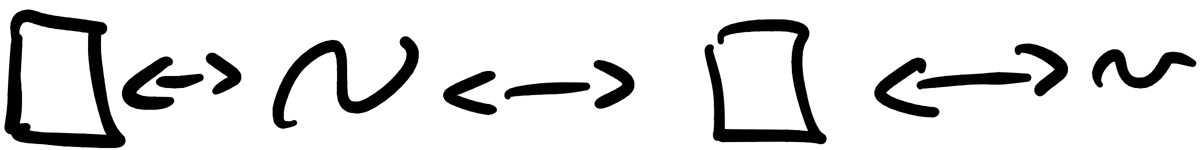
10 μM molecule

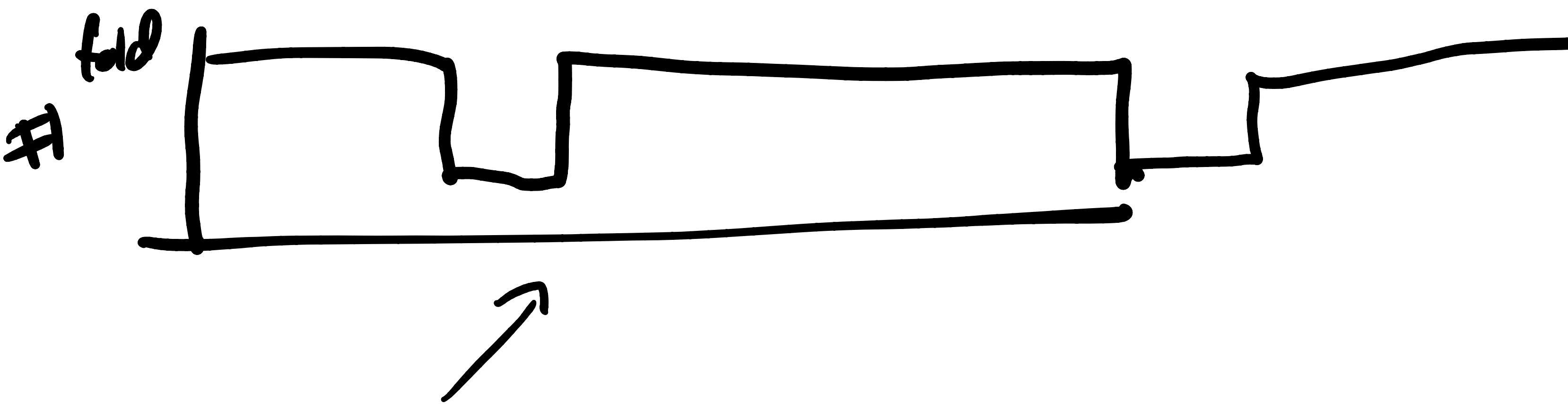
$(100 nm)^3$

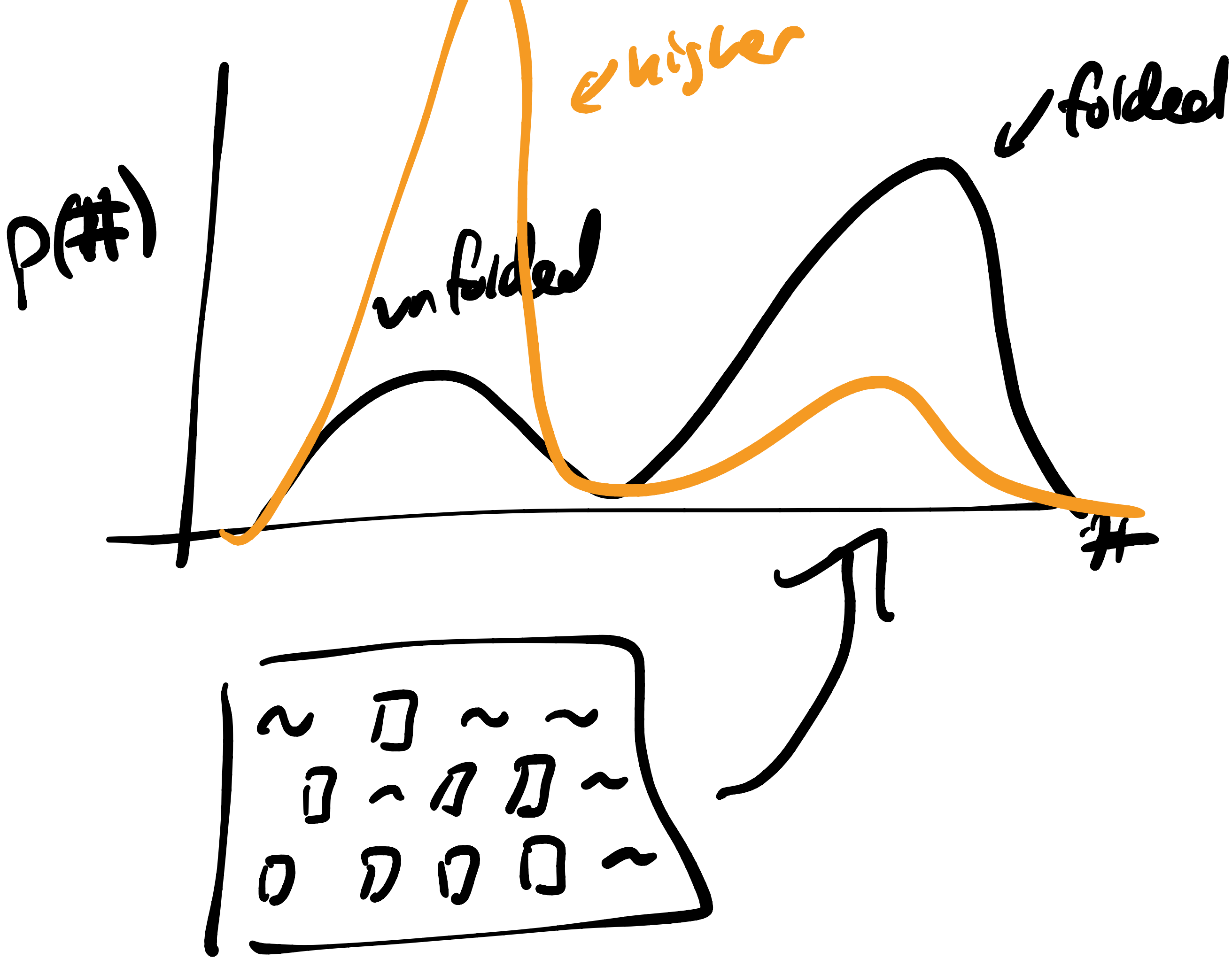
$$\sigma^2 = \mu$$

$$\sigma / \mu \sim \frac{1}{\sqrt{\mu}}$$

Sequence of things vs
ensemble of things






p(#)
unfolded
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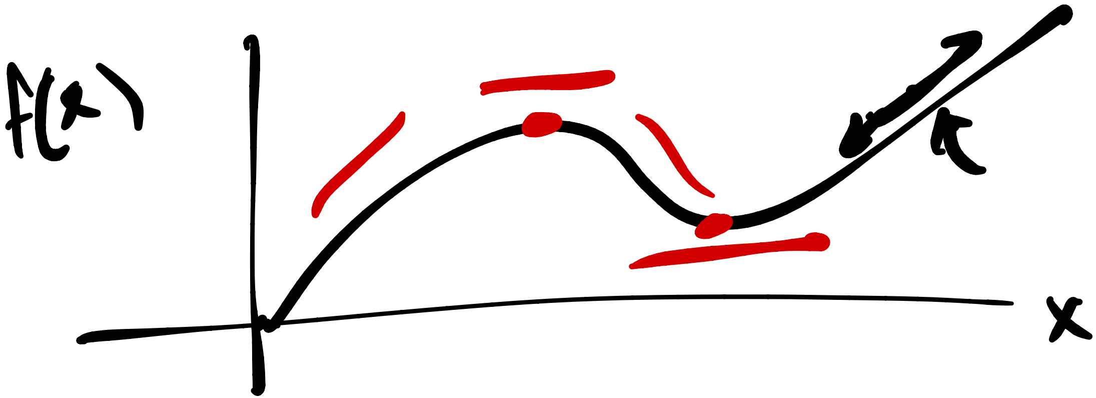
Calculus Quick reminder

(1) derivative is the slope of a function at a point



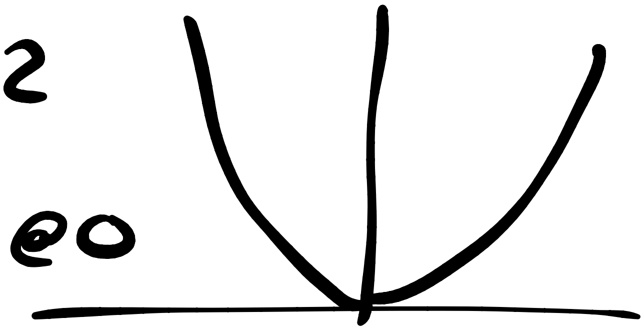
$\frac{df}{dx}$ rate of change

(2) if $\frac{df}{dx} = 0$ max, min, saddle
[check $d^2f/dx^2 > 0$ < 0

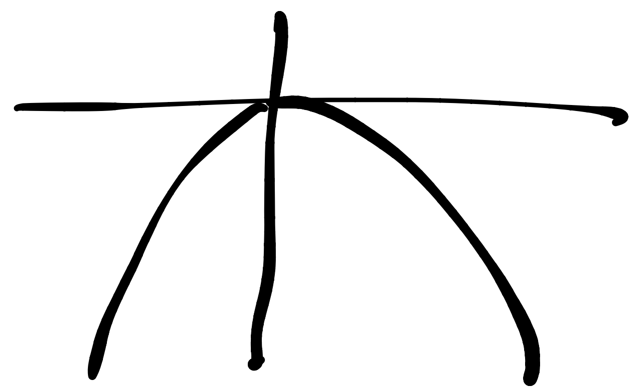
eg $f(x) = \frac{1}{2} k x^2$

$f'(x) = kx$ $= 0$ @ 0

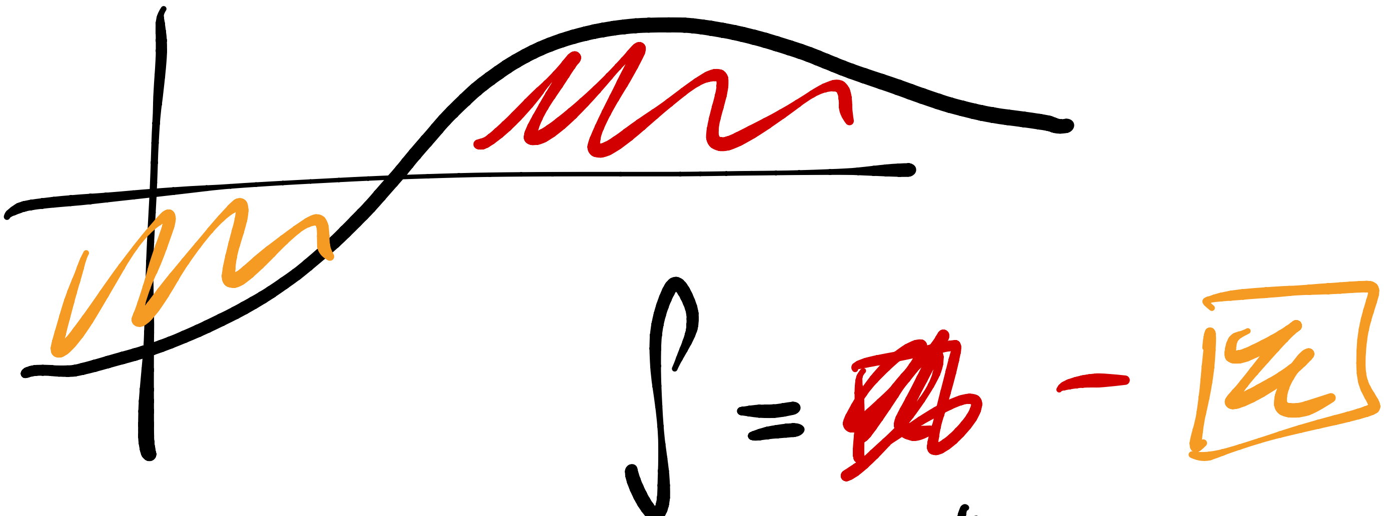
$f''(x) = k > 0$



$f(x) = -\frac{1}{2} k x^2$



③ Integral is area under a curve



④ antiderivative

$$\int_a^b \left[\frac{df}{dx}\right] dx = f(b) - f(a)$$

Derivatives lose some info

$$\int \frac{df}{dx}\,dx = f(x) + C$$

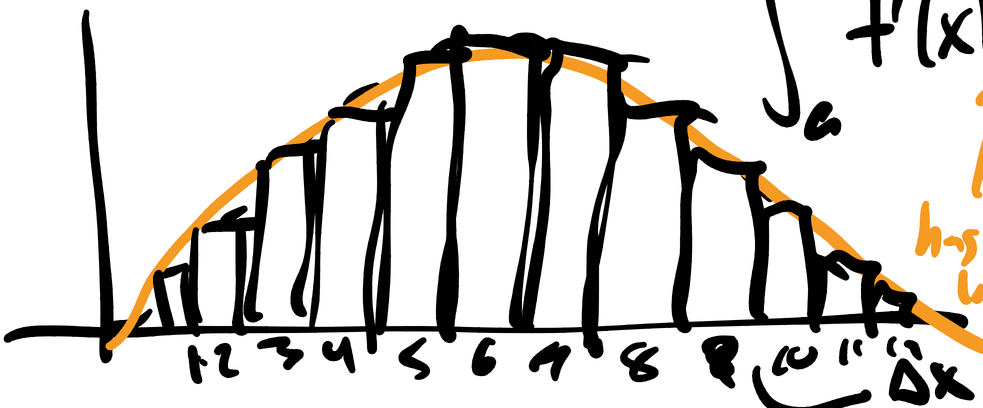
(5) Integral is like a sum

$$\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} f(x_i)\,\Delta x$$

has units

$$\Delta x = \frac{b-a}{N}$$

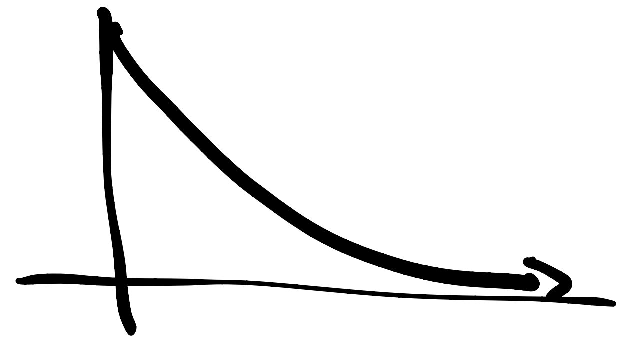
Δx has units



continuous probability distributions

exponential distribution

$$P(t) = \frac{1}{\tau} e^{-t/\tau}$$



$$\langle t \rangle = \int_0^\infty dt \, \frac{1}{\tau} e^{-t/\tau} \cdot t \quad = \tau \, ?$$

$$\sigma^2 = \tau^2$$

Normal distribution

$$P(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu = \mu$

$\sigma^2 = \sigma^2$