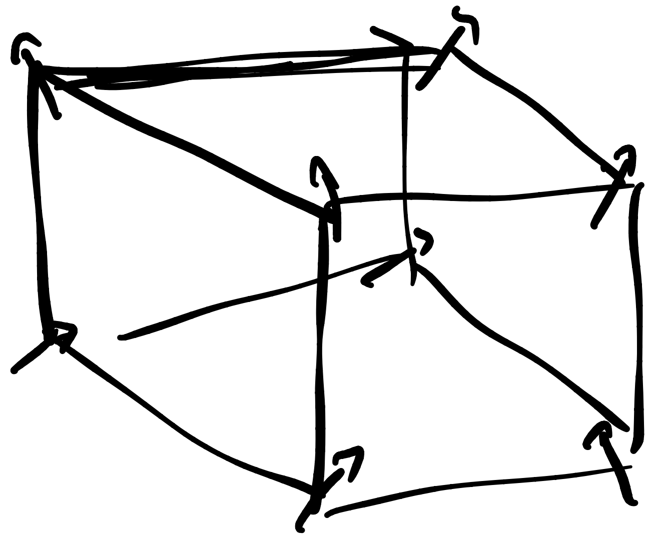


Magnetic:
all spins point
in the same direction

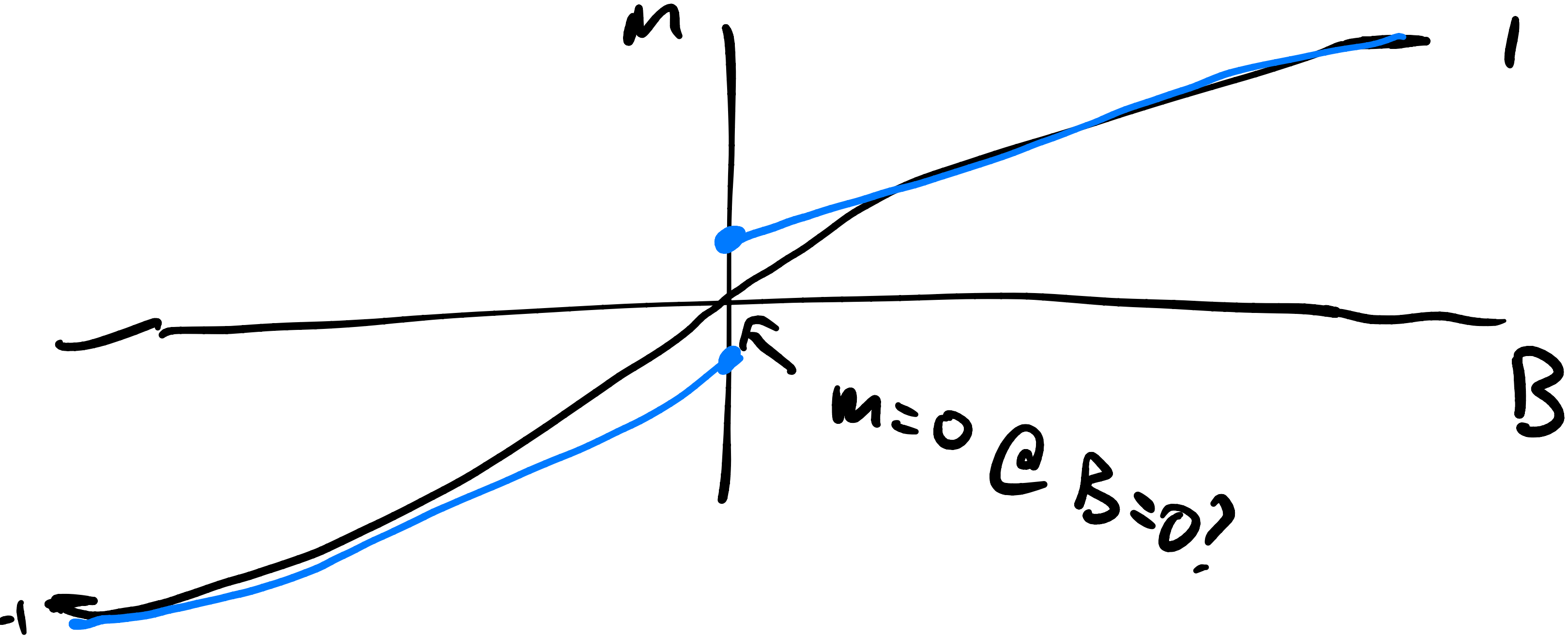
When all point in
same direction?

point field in $\hat{z}$ direction

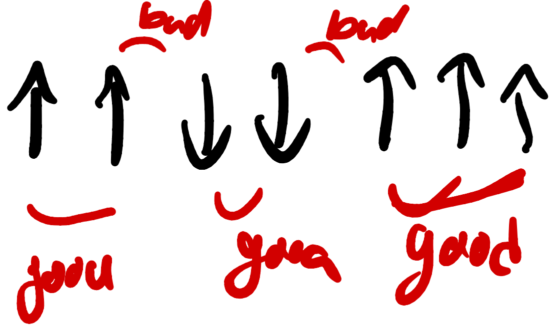
$$M = \sum_{i=1}^{N} \vec{\mu}_i \cdot \hat{z}$$

max is $N\mu$
min is $-N\mu$
if random, $M=0$

$m = M / N\mu$ m goes from -1 to 1

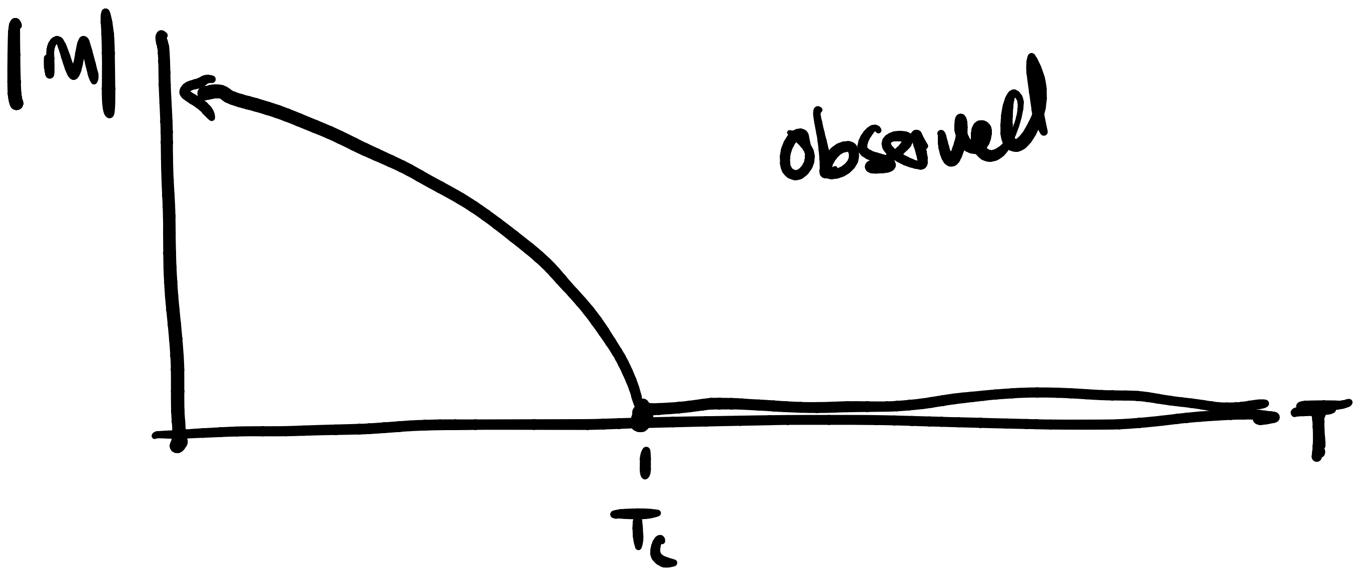


spontaneous magnetization?

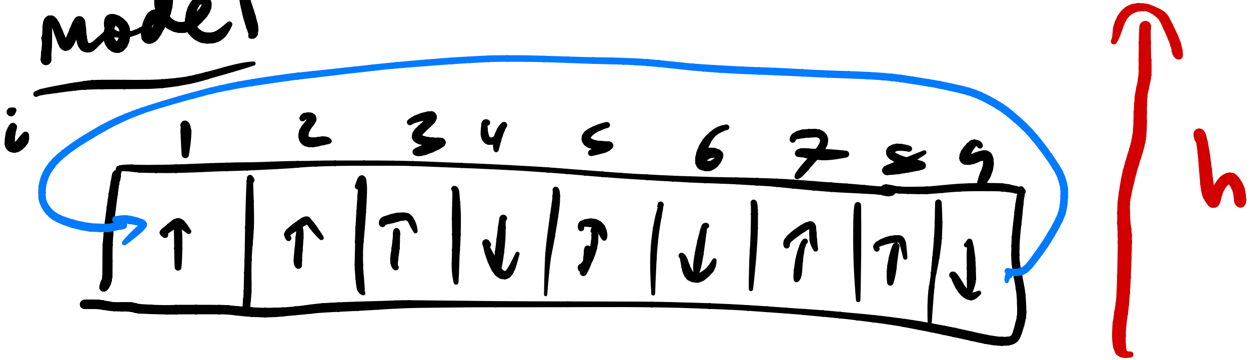


ferro magnetic
spins like to align

if $B = 0$



Ising model



1d chain of spin, point up or down

① $s_i = \pm 1/2$ $+1/2 = \text{up}$ every spin has same spin magnitude

② like to align w/ field

$$\varepsilon_{field} = -h \sum_{i=1}^{N} s_i \qquad \varepsilon \text{ in field } -hs_i$$

like B

(3) neighboring spins like to align

$$\varepsilon^{i}_{neighbor} = -J s_i s_{i-1} - J s_i s_{i+1}$$

$$\varepsilon_{total} = \sum_{i=1}^{N} \left(-J s_i s_{i+1} - h s_i \right)$$

(periodic boundaries)

what is $\langle s_i \rangle = m$

$$M = \sum_{i=1}^{N} S_i = N_{up}\, \tfrac{1}{2} - N_{down}\, \tfrac{1}{2}$$

$$m = M/N = \frac{1}{N} \sum_{i=1}^{N} S_i = \langle S_i \rangle$$

$m(T, h) \leftarrow$ question

higher dimensions?

$$Q = \sum_{\text{micro State}} e^{-\beta E(\text{micro state})}$$

What are microstates?

microstate: $\{ s_1, s_2, \ldots, s_N \}$

$T \to \infty$

$\beta = 1/k_B T \to 0$

$$Q = \sum_{s_1 = \pm 1/2} \sum_{s_2 = +/- 1/2} \cdots \sum_{s_N = +/- 1/2} e^{-\beta E(s_1, s_2 \ldots, s_N)}$$

$$= \sum_{\{s_i\}} e^{-\beta E(s_1 \ldots s_N)}$$

$\beta \to 0$

$$Q = \sum_{s_1} \sum_{s_2} \cdots \sum_{s_N} (1) \qquad \text{counting states}$$

$$= 2 \cdot 2 \cdot 2 \cdot \cdots \cdot 2 = 2^N$$

$$Q = \sum_{\{s_i\}} e^{-\beta \sum_{i=1}^{N} (-J s_i s_{i+1} - h s_i)}$$

$$Q = \sum_{\{s_i\}} e^{-\beta \underbrace{\sum_{i=1}^{N} \left(-J s_i s_{i+1} - h s_i\right)}_{E}}$$

$$e^{\beta \sum J s_i s_{i+1}} \quad e^{\beta h \sum s_i}$$

$$\langle E \rangle = -\frac{\partial \ln Q}{\partial \beta}$$

$$M = \left\langle \sum_{i=1}^{N} s_i \right\rangle = \sum_{\{s_i\}} \left(\sum_{i=1}^{N} s_i\right) \frac{1}{Q} e^{-\beta E(s_1 \ldots s_N)}$$

$$\partial \ln Q / \partial h = \frac{1}{Q} \partial Q / \partial h = \frac{1}{Q} \sum \left(\beta \sum s_i\right) e^{-\beta E(\cdots)} = \beta M$$

$$M = k_B T \frac{\partial \ln Q}{\partial h}$$

$$m = M/N = \frac{k_B T}{N} \frac{\partial \ln Q}{\partial h}$$

@ $T \to \infty$ $\quad Q = 2^N \Rightarrow m = 0$

Spontaneous magnetization

m @ $h = 0$?

$$\lim_{h \to 0} m(h, T) > 0 ?$$

$J = 0$, spins don't see eachother

$$\uparrow \quad \uparrow \quad \downarrow \quad \uparrow \quad \downarrow$$

$$\mathcal{E} = -h\sum_{i=1}^{N} s_i$$

$$Q = \sum_{\{s_i\}} \underbrace{e^{\beta h \sum_{i=1}^{N} s_i}}_{e^{\beta h s_1}\cdot e^{\beta h s_2}\cdots} = \overset{q}{\overset{\shortparallel}{\left(\sum_{s_1} e^{\beta h s_1}\right)}} \overset{q}{\left(\sum_{s_2} e^{\beta h s_2}\right)} \cdots \underset{q}{\left(\sum_{s_N} e^{\beta h s_N}\right)}$$

$$Q = q^N \quad (\text{independent})$$

$$+h/2 \;—\; \varepsilon_2$$

$$-h/2 \;—\; \varepsilon_1$$

$$q = e^{\beta h \cdot 1/2} + e^{\beta h \cdot -1/2}$$

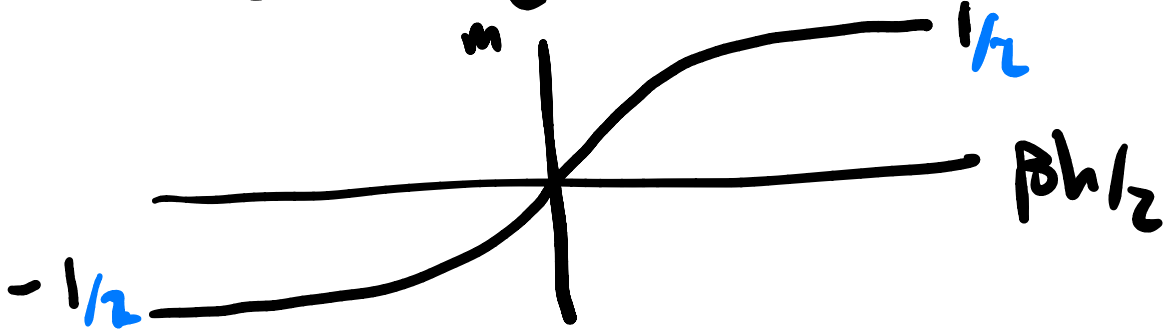
$$Q = \left(e^{\beta h/2} + e^{-\beta h/2}\right)^N$$

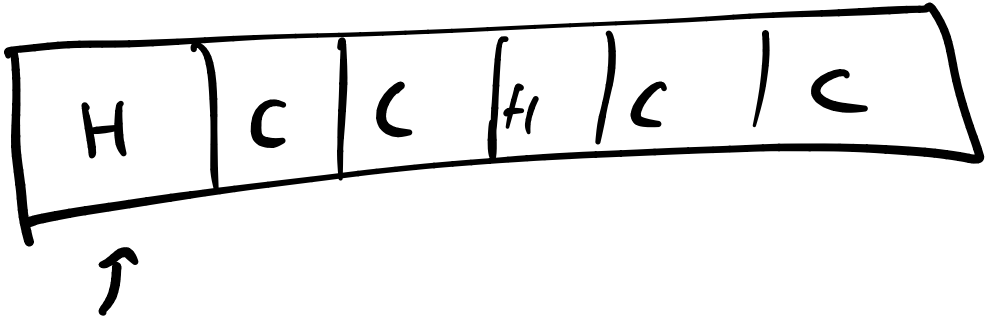
$$m = \frac{k_B T}{N} \frac{\partial \ln Q}{\partial h} = k_B T \frac{\partial \ln\left(e^{\beta h/2} + e^{-\beta h/2}\right)}{\partial h}$$

$$m = k_B T \cdot \frac{1}{q} \, {\partial q}/{\partial h} \qquad q = e^{\beta h/2} + e^{-\beta h/2}$$

$$\frac{\partial q}{\partial h} = \beta/_2 \, e^{\beta h/2} - \beta/_2 \, e^{-\beta h/2}$$

$$= \frac{1}{2} \cdot \frac{\left(e^{\beta h/2} - e^{-\beta h/2}\right)}{e^{\beta h/2} + e^{-\beta h/2}} = \frac{1}{2} \tanh(\beta h/2)$$





entropic penalty

$+\epsilon_H N_H$ $\quad -\epsilon_H$ good to be helical

$-\epsilon_{HH} N_{H-H\text{'s}}$

$$q = \underline{1} + \underbrace{e^{\beta\epsilon}}_{K} \qquad P_H / P_C$$

no coupling

$$P_H = e^{\beta\epsilon} / 1+e^{\beta\epsilon} \qquad P_C = 1 / 1+e^{\beta\epsilon}$$

$$q = 1 + K \qquad q^N = (1+K)^N$$

$$\xi^4 = 1 + 4k + 6k^2 + 4k^3 + k^4$$

$$\binom{N}{m}$$

$$Q = \sum_{m=0}^{N} \binom{N}{m} k^m$$

2^N states