Probability overview

Preview:

$$S = k_B \ln(\#\text{ states of system})$$

[constant energy]

Random walk - brownian

<u>Independent events</u>

flipping (fair) coin

probability of an outcome doesn't depend on anything (previously)

Coin flipping: H, T

H	T
50%	50%

100%

Exclusive outcomes

probabilities add to 1

$$1 = P_H + P_T$$

6 sided die

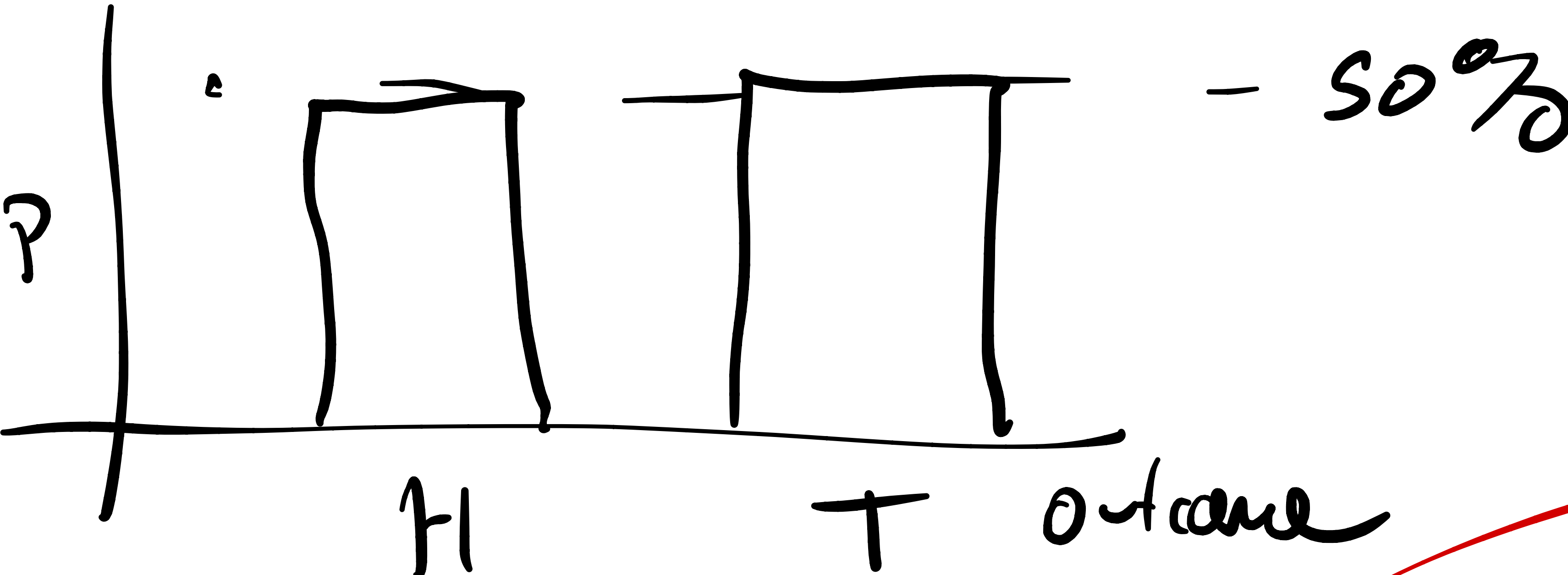
1	2	3
4	5	6

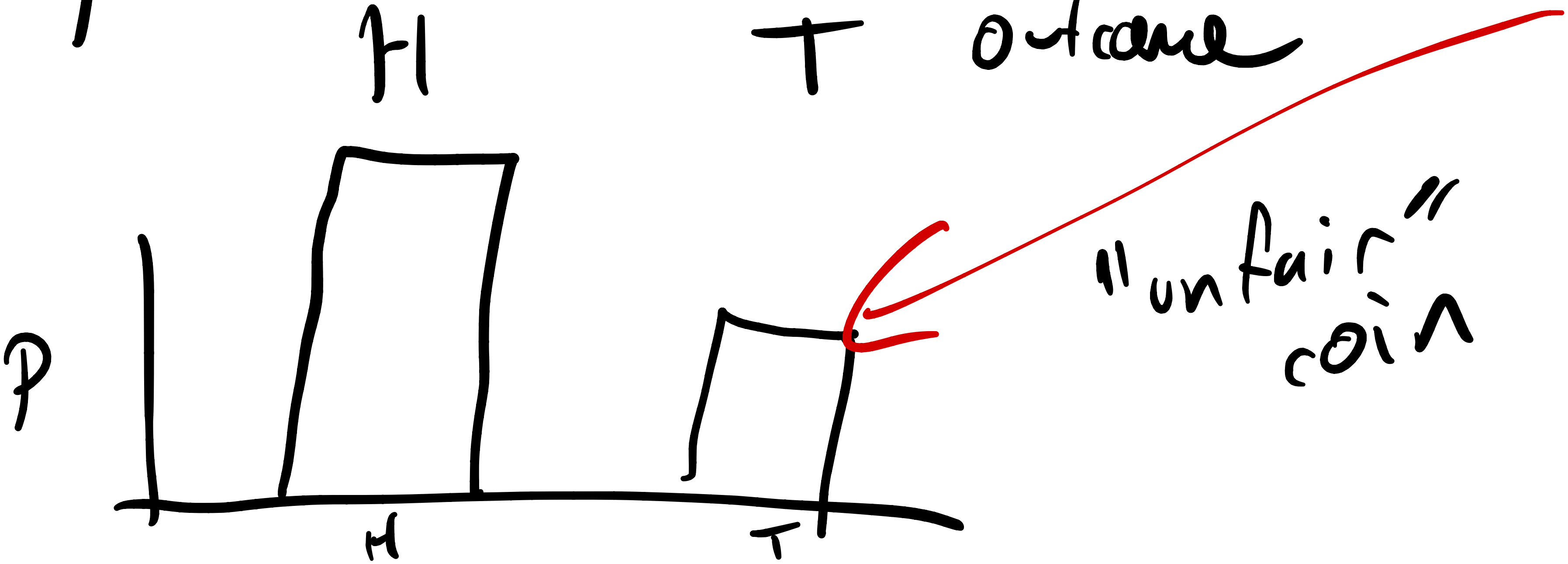
$$1 = P_1 + P_2 + \dots + P_6$$

Distribution

lots of times
(bar chart)

coin

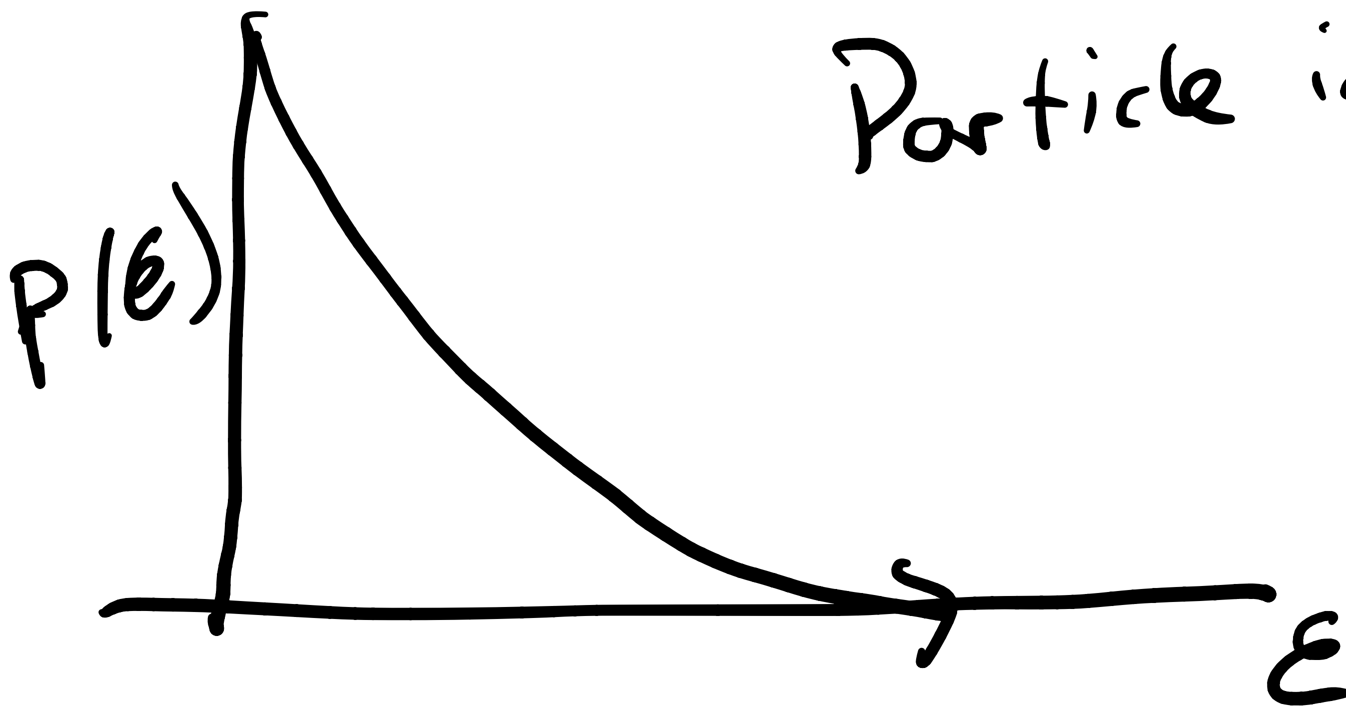




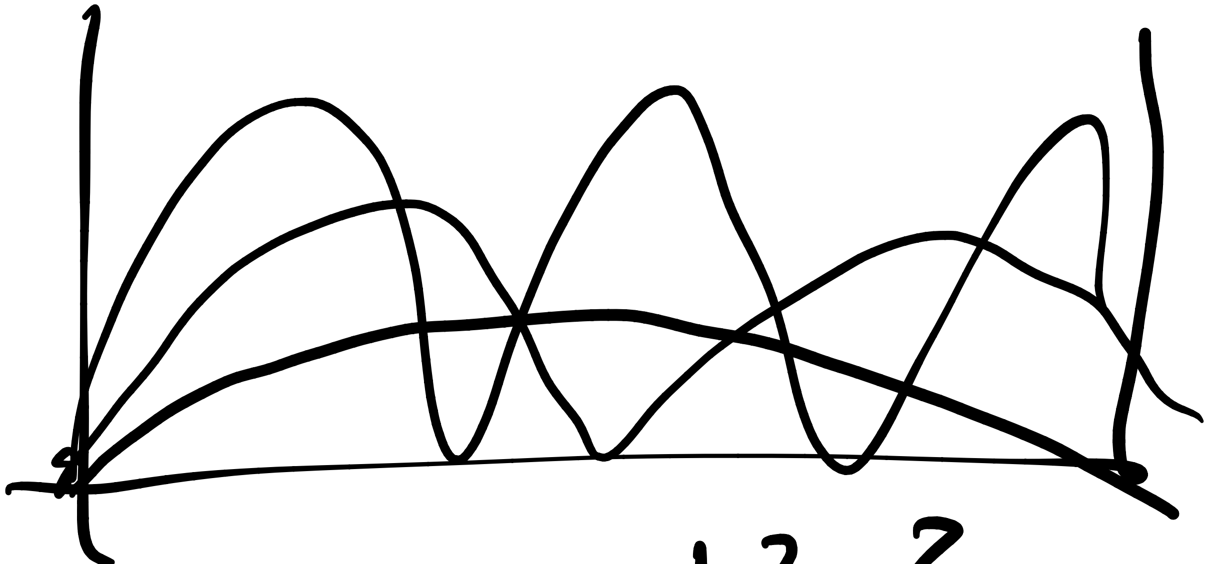
Later Temperature T

prob of molecule being in state

$$P(\mathcal{E}_{state}) \propto e^{-\mathcal{E}/k_B T}$$

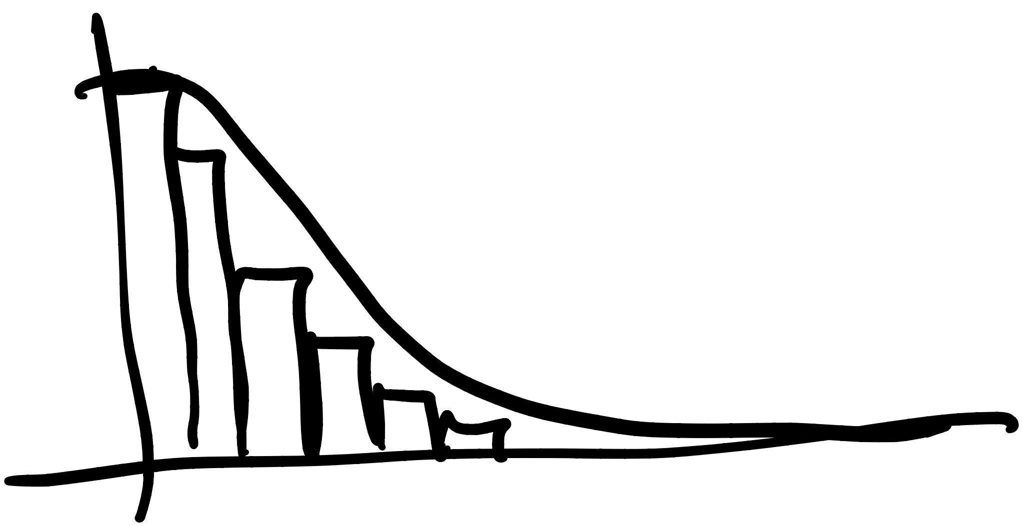


Particle in a box



$$\varepsilon_n = \frac{h^2 n^2}{8 m L^2}$$

$$e^{-\varepsilon/k_B T}, \quad e^{-n^2}$$



$$1 = \sum_{n=1}^{\infty} P(n)$$

Combine probabilities

"Logical operation"

And "$\cap$" $\wedge$

Or "$\cup$" $\vee$ and/or

or but not both

Single event

1	2	3
4	5	6

$2/6 = 1/3$

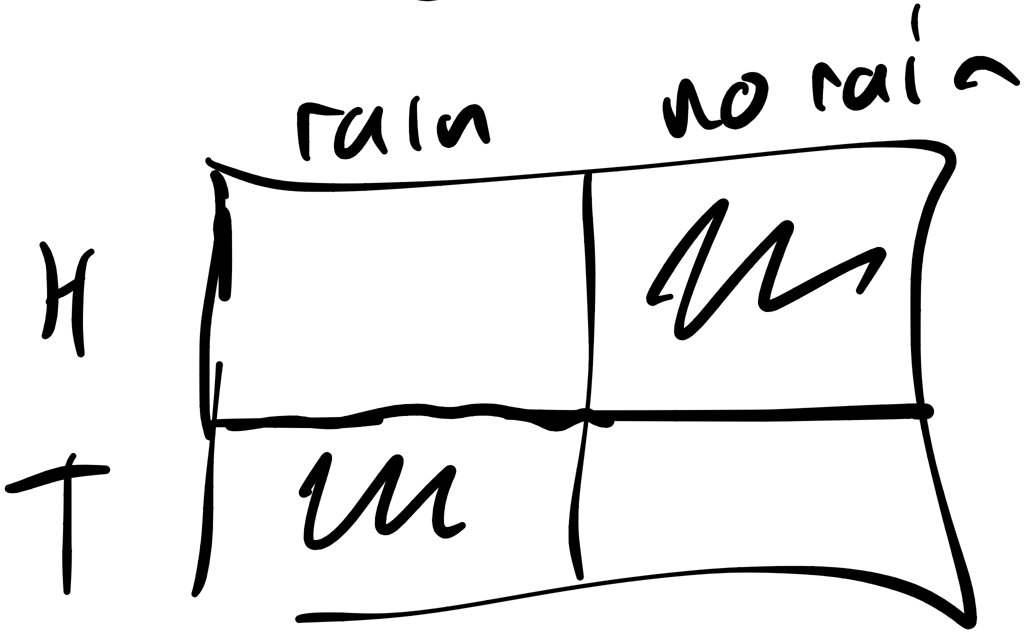
$P_1 \cap P_2 = 0$

$P_1 \cup P_2 = P_1 + P_2$

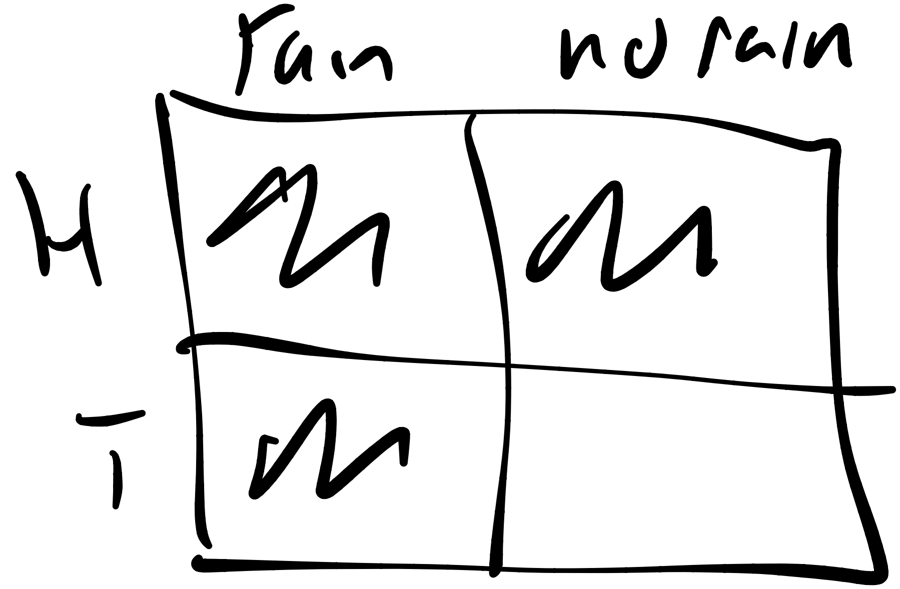
$P_{rain} \cup P_{Head}$

H $\quad \frac{1+1}{4}$

$\frac{1+1+1}{4}$



exclusive or



and + or

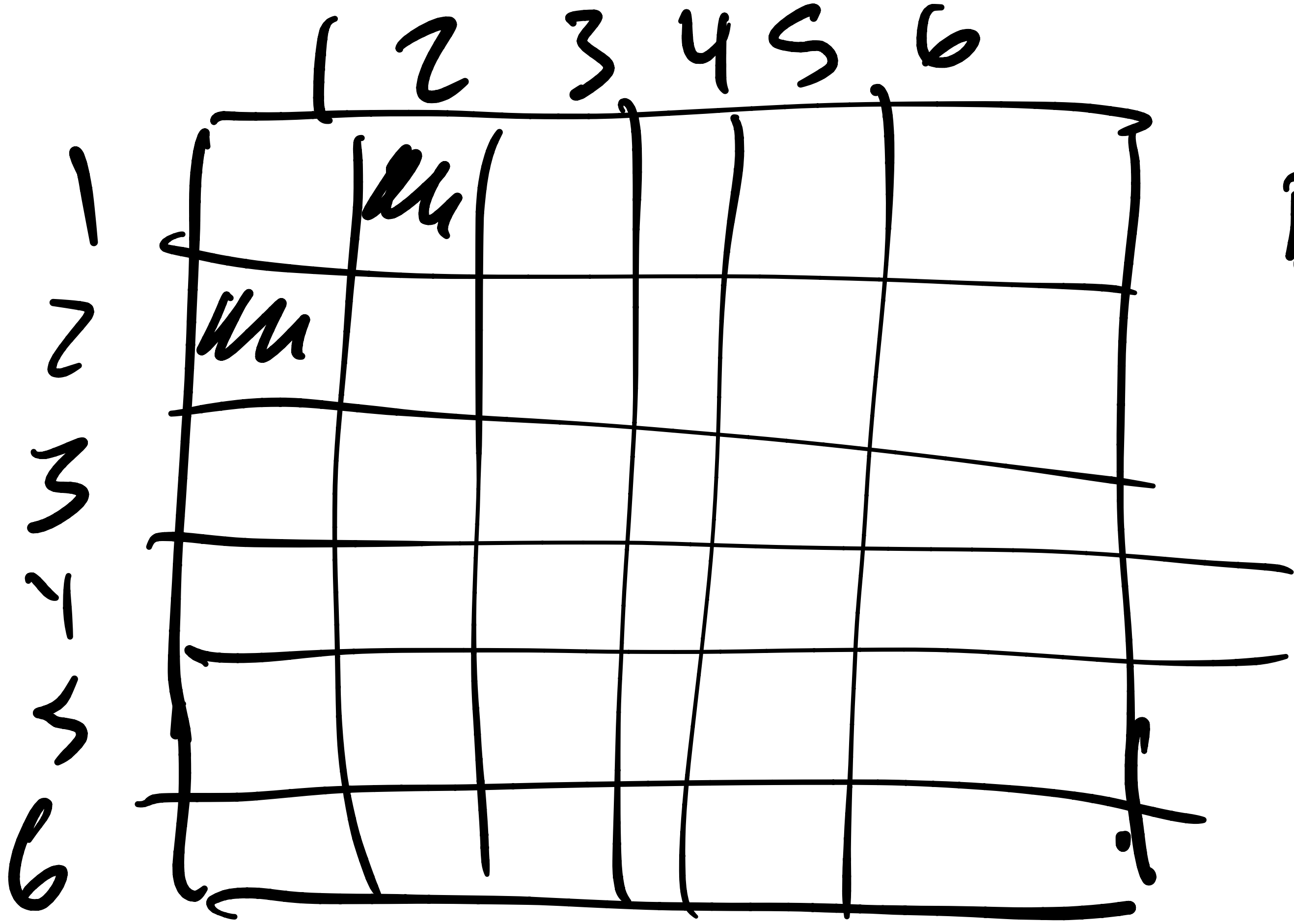
$P_{rain} \cap P_{Head}$

for single event

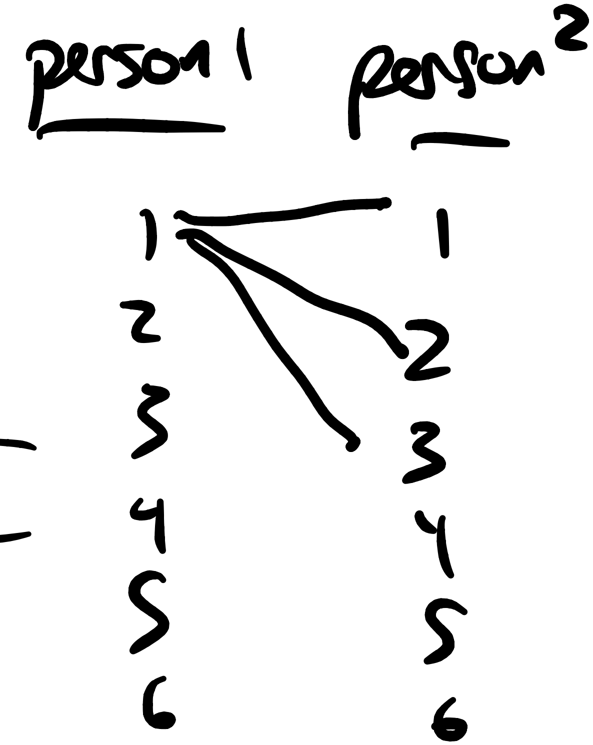
Probability of independent events

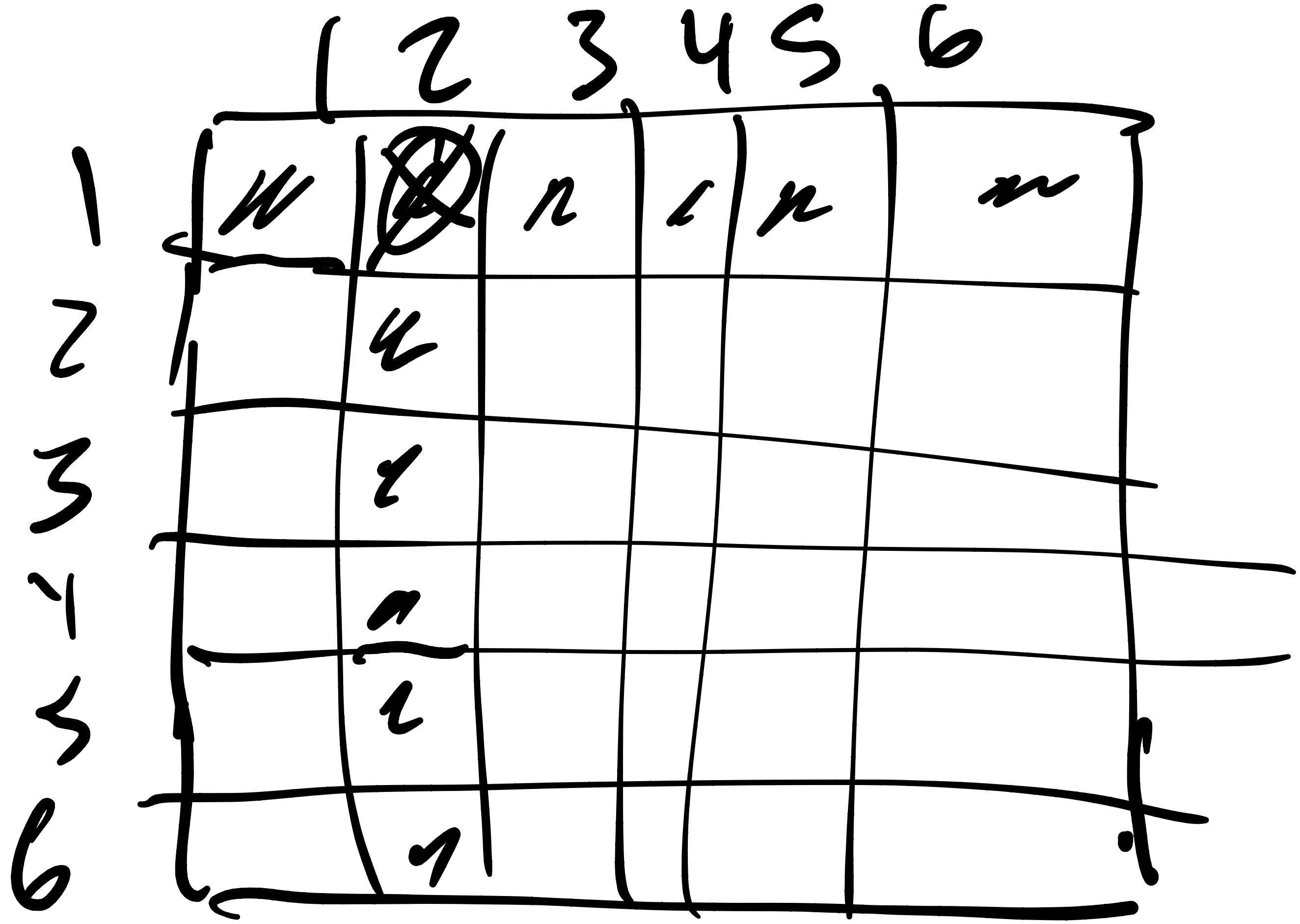
roll a dice twice

$P_1 \cap P_2$



2/36



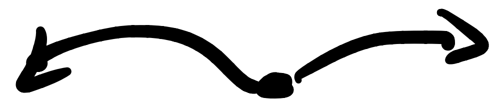


roll 1 is a 1

roll 2 is a 2

Sequence of events

coin flip (random walk 1d



flip a coin N times
(5)

_ _ _ _ _

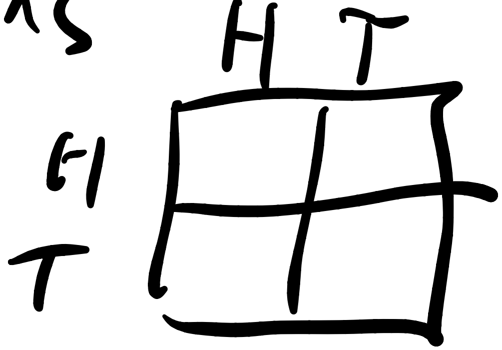
H H H H H
H H H H T
H H H T H
H H H T T
H H T H H
H H T H T
H H T T H
H H T T T

4

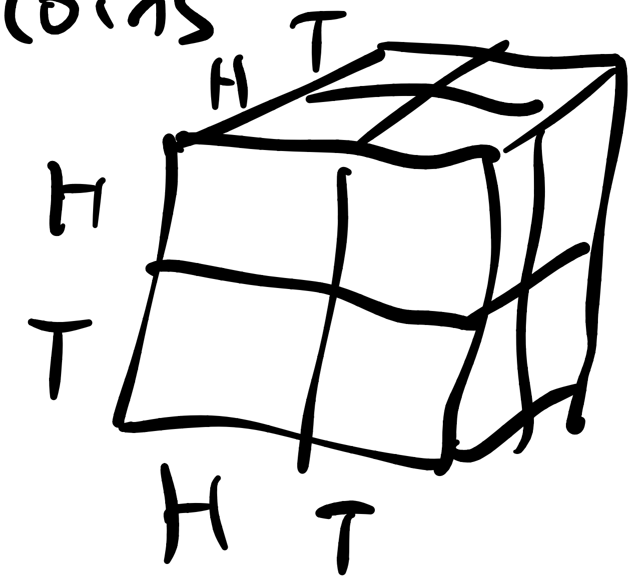
H T H H H
H T H H T
H T H T H
H T H T T
H T T H H
H T T H T
H T T T H
H T T T T

+ another 16 w/ T first

2 coins



3 coins



$2, \quad 2 \times 2, \quad 2 \times 2 \times 2$

In general 2^N

Particular sequence

Eg $P_{HHTTT} = \frac{1}{2^5}$

$P_{H \cap H \cap T \cap T \cap T} = P_H \cdot P_H \cdot P_T \cdot P_T \cdot P_T$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

Prob of Exactly 2H (& 3T) in 5 events

What is number of ways to order N things $= N!$ (distinguishable)

$\underbrace{ABC}_{3\ \text{thing}}$

ABC
ACB
BAC
BCA
CAB
CBA

3 things $\cdot$ 2 things $\cdot$ 1 thing

$\underbrace{3 \cdot 2 \cdot 1}_{3!}$

5 coin outcomes

___ ___ ___ ___ ___

2 H H_1 H_2 H H H H_2 H

3 H H_1 _ H_2 _ _ H_2 H_1

H_1 _ _ H_2 _ H_2 H_1

H_1 _ _ _ H_2 H_2 H_1

$5 \cdot 4 \cdot 3$

m thing in N slots

$$\frac{N!}{(N-m)!} = N \cdot (N-1) \cdot (N-2) \cdots (N-m)$$

but if indistinguishable

ways to arrange m things

$m!$

$$\frac{N!}{(N-m)!(m)!} = \binom{N}{m}$$ N choose m

$$\frac{N!}{(N-m)!\,(m)!} = \binom{N}{m}$$

$$N_H + N_T = N$$

m

$$N_T = N - N_H$$

Same formula for choosing 2 H or 3 T in 5 slots