

Last time

$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, P, n_{j \neq i}}$$

Molecules convert from high μ state
to a low μ state, until $\mu_i = \mu_j$

At const T & P

$$\mu_i = \bar{G}_i = G_i / n_i$$

$$G = \sum n_i \mu_i$$

how does μ depend on T & P ?

Consider within one phase

$$\mu = \bar{G} = \bar{H} - T\bar{S}$$

$$G = H - TS$$

How does H & S depend
on T & P

$$dH = d(E + pV) = dE + \underline{pdV} + \underline{Vdp}$$

$$dE = dq - \underline{pdV}$$

+

@ const T & P

$$dH = dq = C_p dT$$

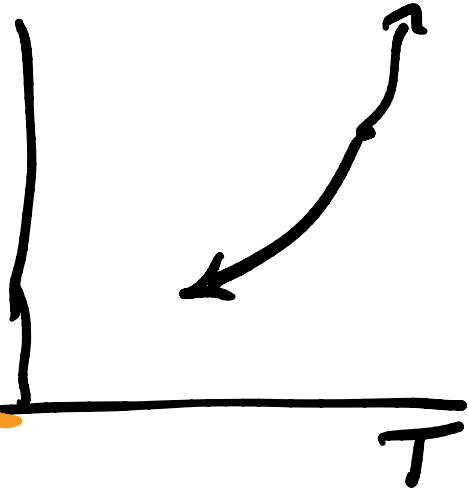
$$T ds = dq \quad \int$$

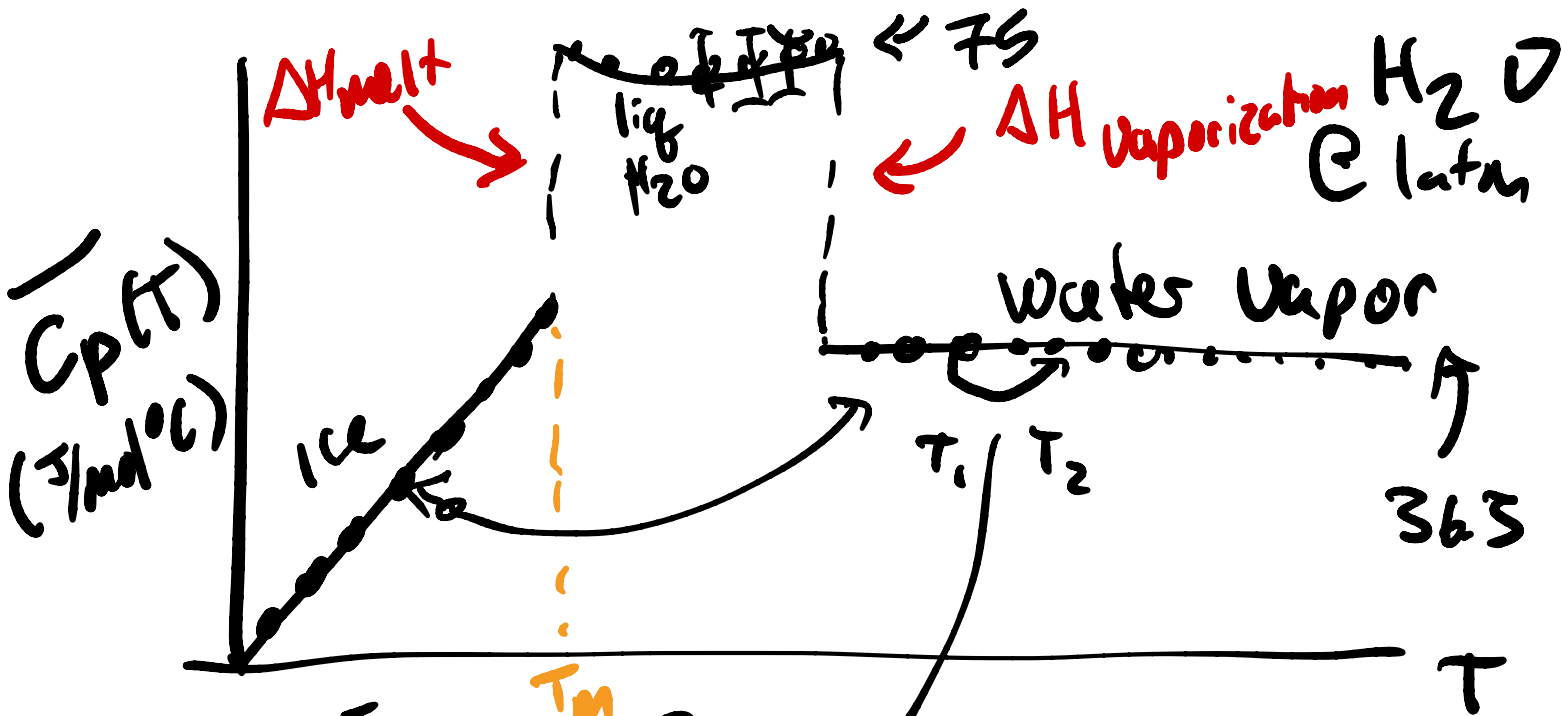
$$dH = C_p dT$$

$$ds = C_p/T dT$$

need
to integrate

need to know C_p





$$\Delta H = \int_{T_1}^{T_2} dH = \int_{T_1}^{T_2} C_p(T) dT \approx C_p^{gas} \Delta T$$

$$\Delta \bar{H} = \bar{C}_p^{gas} \Delta T$$

$$\bar{H}(T) = \bar{H}(T_{\text{ref}}) + \int_{T_{\text{ref}}}^T \bar{C}_p dT$$

$$\bar{S}(T) = \bar{S}(T_{\text{ref}}) + \int_{T_{\text{ref}}}^T \frac{\bar{C}_p}{T} dT$$

T_{ref} for H is T_{melt} , $\bar{H}(T_m) = 0$ ↙ solid phase

T_{ref} for S , $\bar{S}(T_{\text{ref}} = 0) = 0$

$$\Delta S_{\text{melt}} \quad \Delta S_{\text{vap}}$$

$$\Delta \bar{G} = \bar{G}(\text{liq}, T_m) - \bar{G}(\text{solid}, T_m) = 0$$

@ T_m liq & solid are in equilibrium

$$\mu_{\text{liq}} = \mu_{\text{solid}}$$

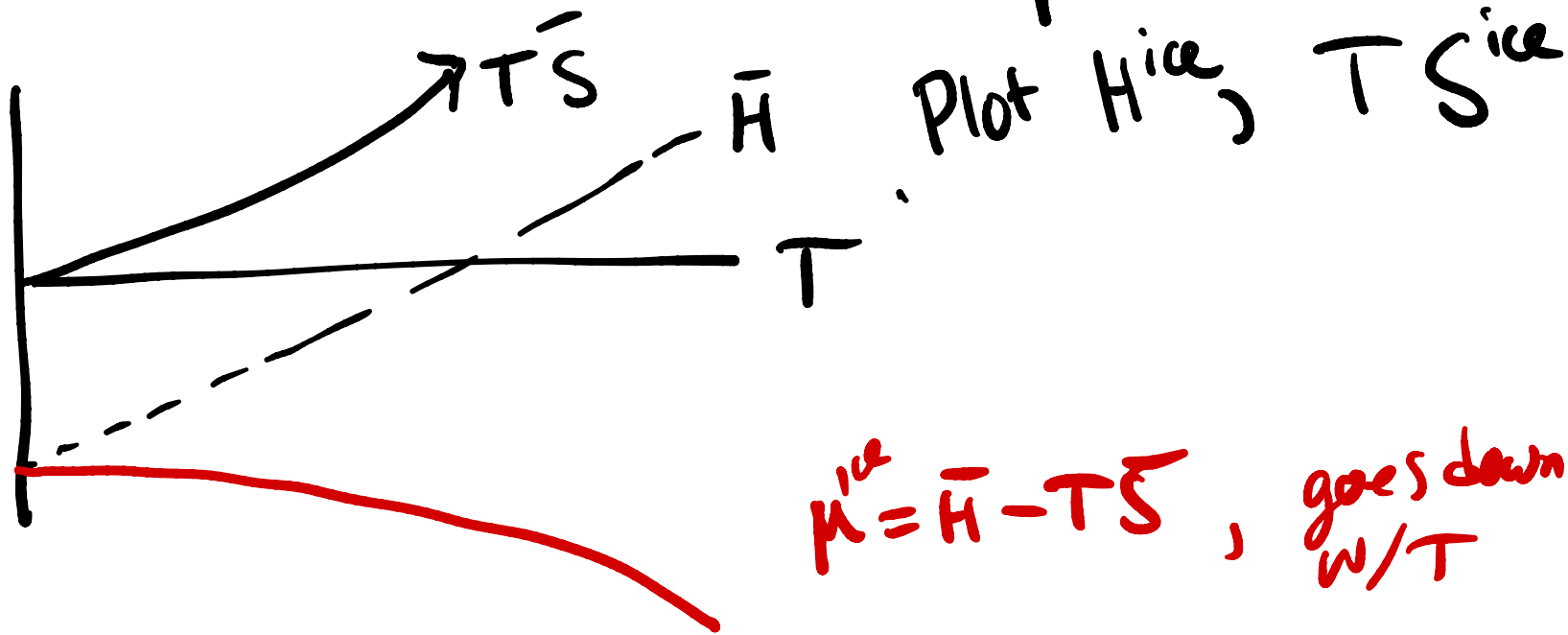
$$0 = \Delta \bar{H}_{\text{melt}} - T_m \Delta \bar{S}_{\text{melt}}$$

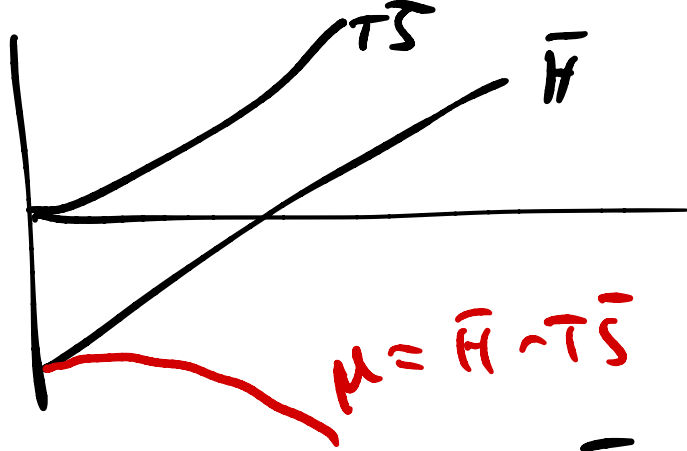
$$\Delta \bar{S}_{\text{melt}} = \frac{\Delta \bar{H}_{\text{melt}}}{T_m}$$

↙ Kelvin

$$\Delta \bar{H}_m = \bar{H}^{\text{liq}}(T_m) - \bar{H}^{\text{sol}}(T_m)$$

have functions fit for $\bar{C}_p(T)$





$$(4) \left(\frac{\partial \mu}{\partial T} \right)_P = -S \text{ decreasing}$$

(1)

$$@ T=0 \quad \mu = \bar{H}$$

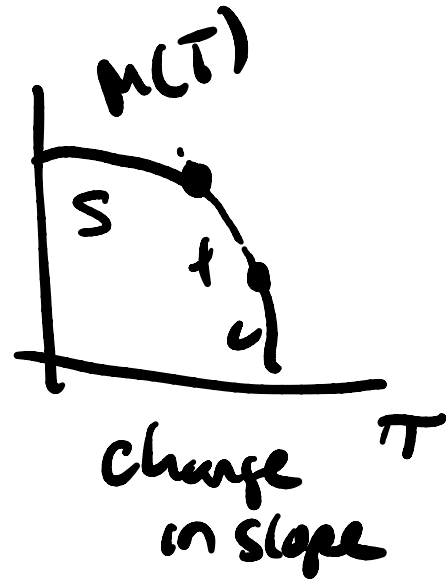
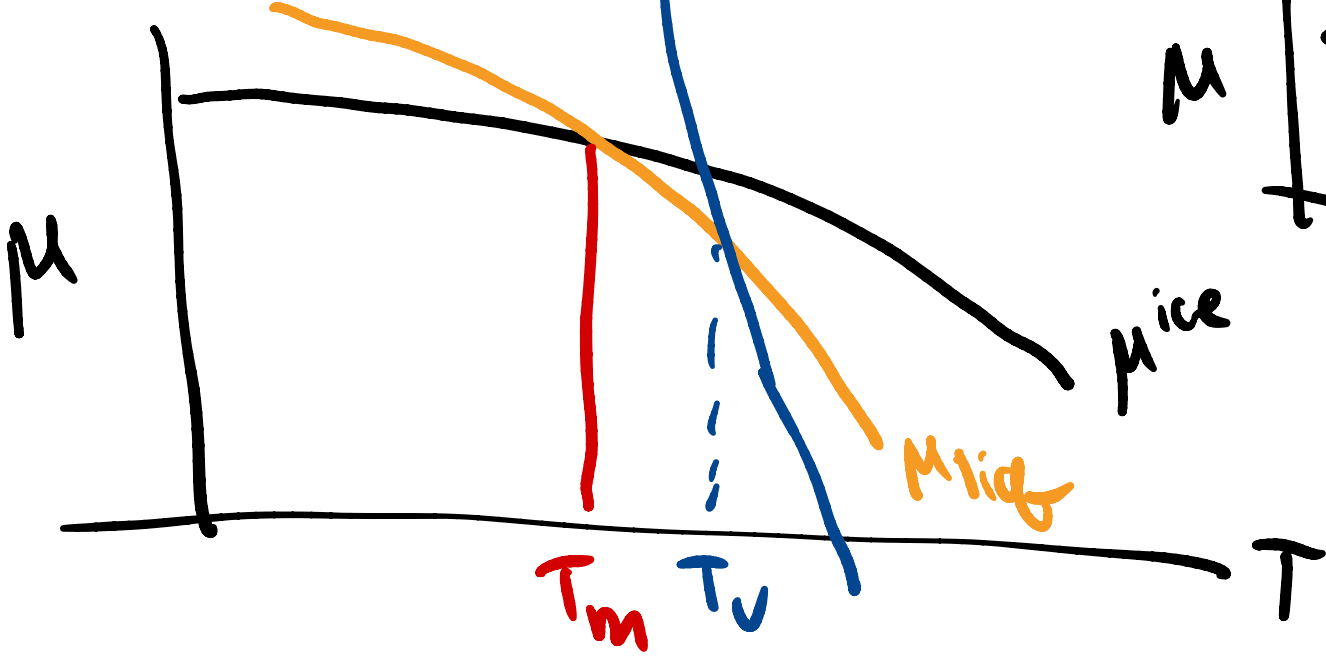
(2)

$$\bar{C}_P^{ice} = \left(\frac{\partial H^{ice}}{\partial T} \right)_P > 0 \text{ every where}$$

(3)

$$\left(\frac{\partial S}{\partial T} \right)_P = \bar{C}_P / T > 0 \quad S \text{ increases with } T$$

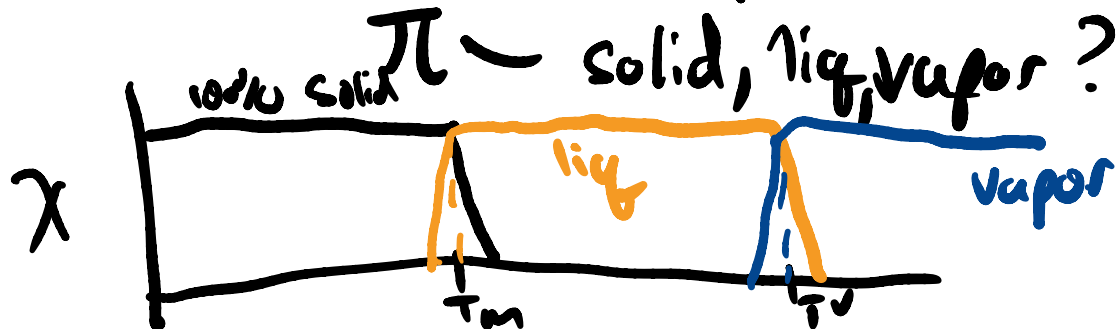
Ice chemical potential going down w/T!



If small (nano scale) system
no true true phase transitions

$$G^{\text{system}} = \frac{n_{\text{solid}}}{n_{\text{tot}}} \mu_{\text{solid}} + \frac{n_{\text{liq}}}{n_{\text{tot}}} \mu_{\text{liq}} + \frac{n_{\text{vap}}}{n_{\text{tot}}} \mu_{\text{vap}}$$

$$= \sum X^{\pi} \mu^{\pi}$$



How does pressure affect μ

P biggest effect on gasses

Gibb-Duhem

$$C_s \quad d\mu^\pi = -\bar{S}^\pi dT + \bar{V}^\pi dP$$

@ const T

$$= \bar{V}^\pi dP$$

how does \bar{V} in liq, solid or gas change

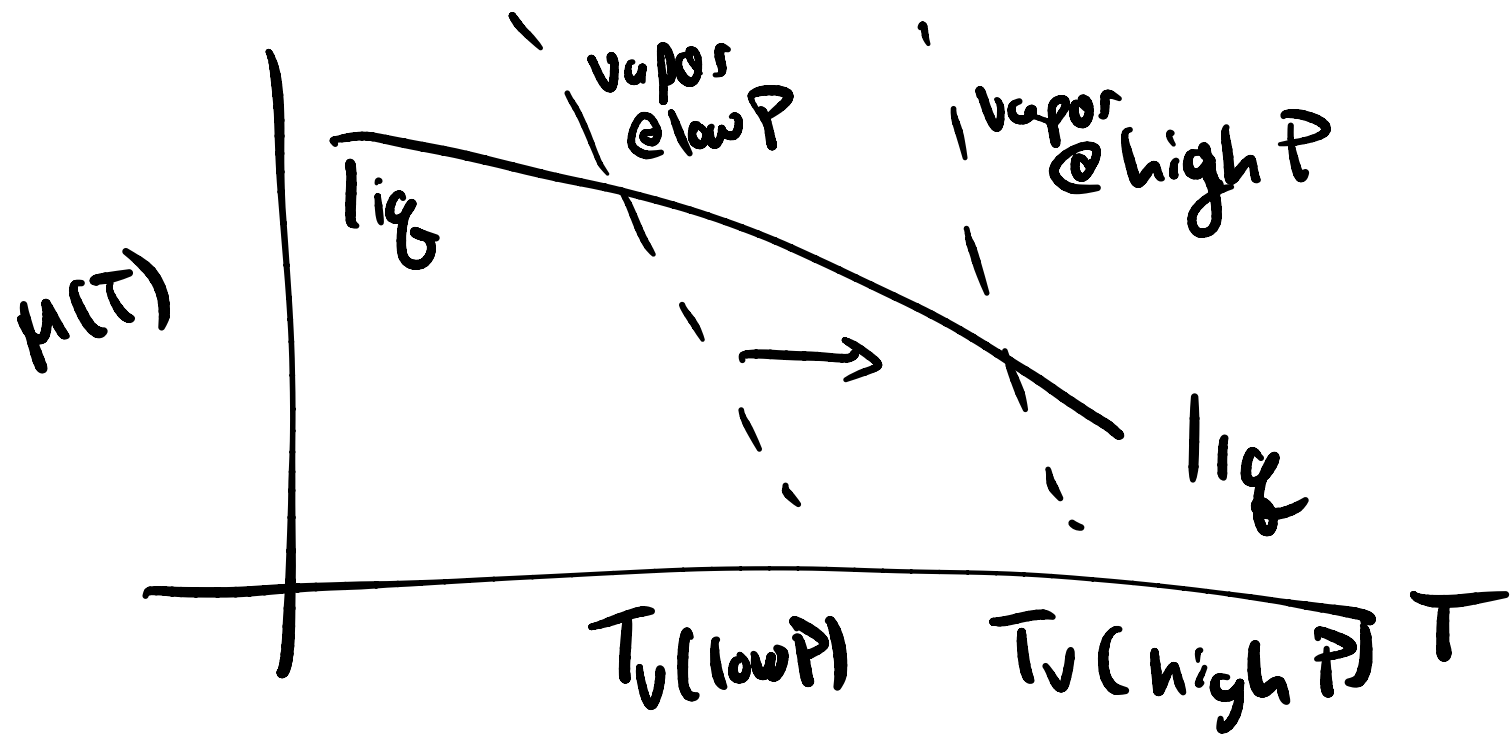
$$d\mu^{\text{gas}} = \bar{v} dp \approx \frac{RT}{P} dP$$

$$\Delta\mu = \int_{1\text{atm}}^P d\mu = \int_{1\text{atm}}^P \frac{RT}{P} dP$$

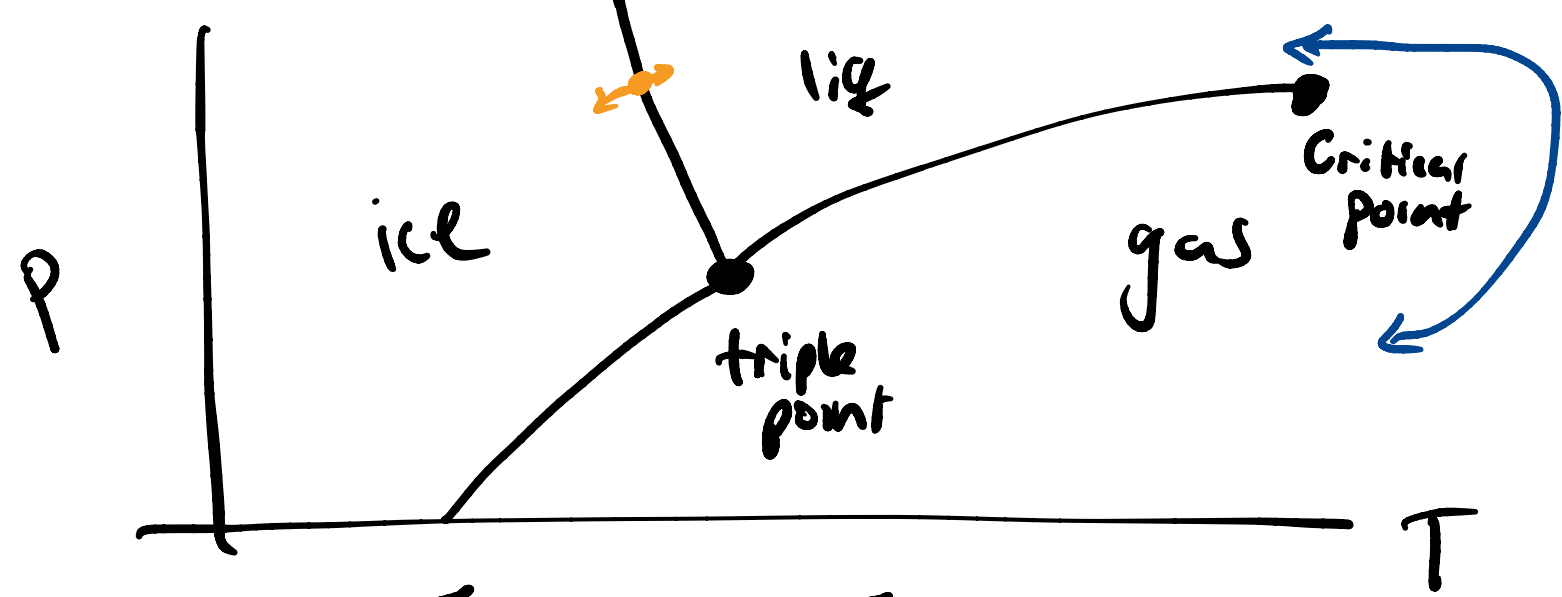
$$= RT \ln(P/1\text{atm})$$

$$PV = nRT$$

$$V/n = \frac{RT}{P}$$



boiling point elevation

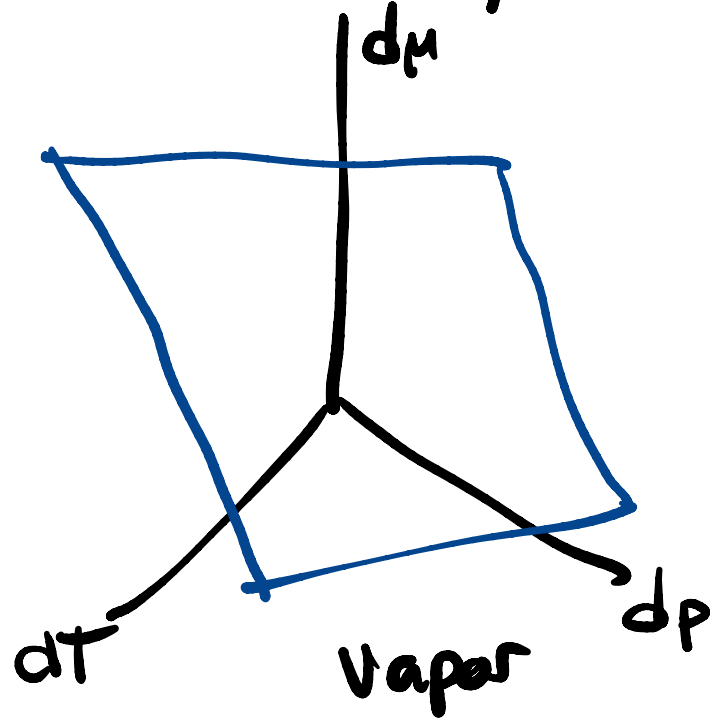
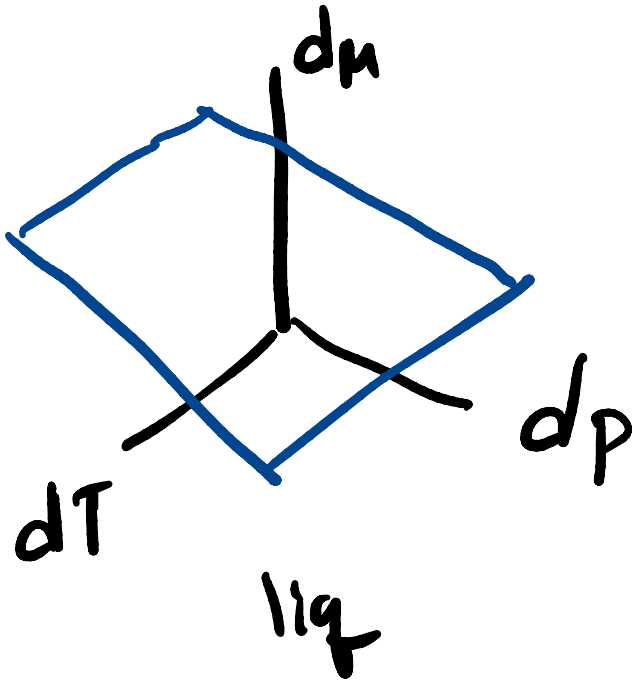


line: $\mu^{\pi_1}(T, P) = \mu^{\pi_2}(T, P)$

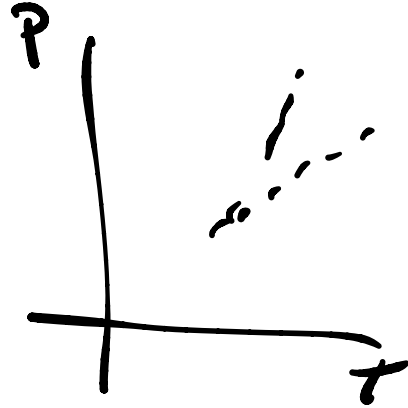
T.P. $\mu^{\text{ice}} = \mu^{\text{water}} = \mu^{\text{gas}}$, $5 \times 10^8 \text{ atm}$

$$GD \text{ eqn } d\mu + \bar{S}dT - \bar{V}dp = 0$$

one equation for μ in each phase



2 planes of $\mu(T, P)$
intersect along a line



Can show

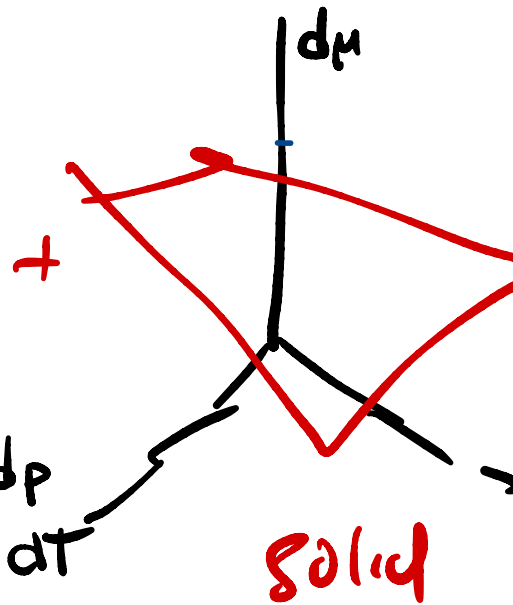
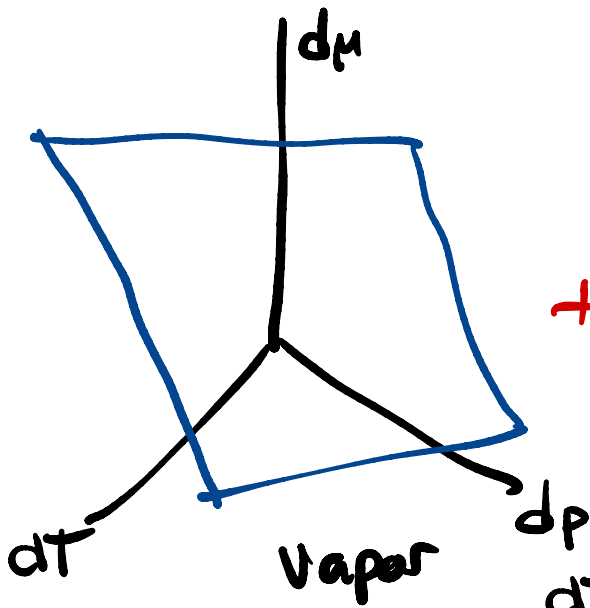
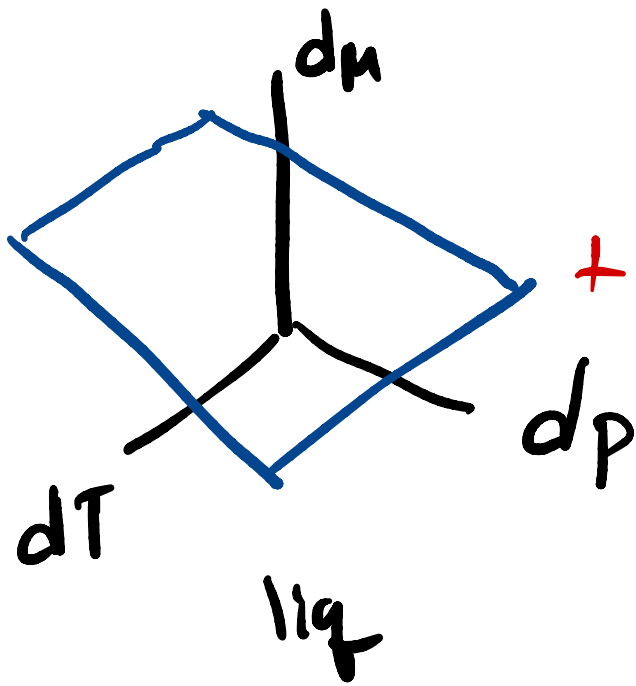
$$\left(\frac{\partial P}{\partial T}\right)_{\mu_{\text{phase 1}} = \mu_{\text{phase 2}}} = \frac{\Delta S^{1 \rightarrow 2}}{\Delta V^{1 \rightarrow 2}}$$

> for almost everything

eg $l \rightarrow \text{vap}$

Clausius - Clapyron equation

H_2O exception



3 planes can intersect only at a point

Gibbs phase rule

$$\text{d.o.f.} = \# \text{ components} - \# \text{ coexisting phases} + 2$$

things you can change & still coexist

1 component	<u>#</u>	<u>d of</u>	
	1 phase	2	P, T
	2 phases	1	Change T also change P