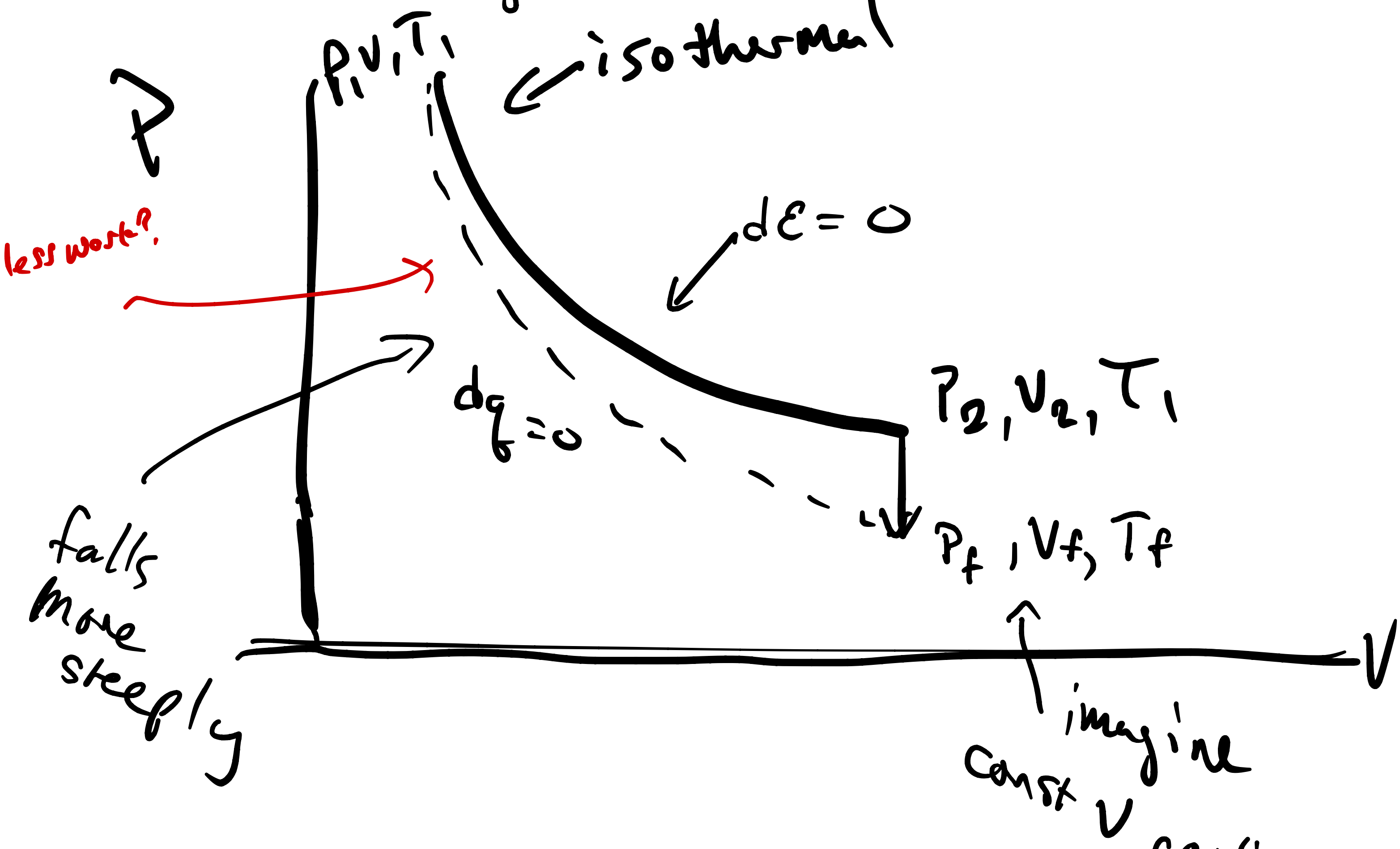
Cont, adiabatic expansion ideal gas

$$dE = dq_{r} + dw = dw$$

$$dw = -P\,dV \underset{\text{ideal gas}}{=} -\frac{nRT(V)}{V}\,dV$$



So ΔE total $= 0 + C_v \Delta T$

or $dE = C_v\,dT$

$$\Rightarrow \frac{C_v\,dT}{T} = -\frac{nR}{V}\,dV$$

$$\Rightarrow C_v \ln\left(T_f/T_i\right) = -nR\ln\left(V_f/V_i\right)$$

$$\Rightarrow T_f/T_i = \left(V_i/V_f\right)^{nR/c_v}$$

$$P = nRT/V \quad \Rightarrow \frac{P_f V_f}{nR} = T_f \quad \text{etc}$$

$$\Rightarrow \frac{P_f V_f}{P_i V_i} = \left(V_i/V_f\right)^{nR/c_v}$$

$$\Rightarrow P_f/P_i = \left(V_i/V_f\right)^{nR/c_v + 1}$$

$$= \left(V_i/V_f\right)^{c_p/c_v}$$

$$\gamma \equiv c_p/c_v$$

During expansion

$$P(V) = P_i V_i^{\gamma}/V^{\gamma} = \lambda/V^{\gamma} = \lambda V^{-\gamma}$$

$\lambda \leftarrow$ const

$$\text{so} \quad W = -\int_{V_i}^{V_f} P\,dV = -\lambda \frac{V^{-\gamma+1}}{-\gamma+1} = \frac{\lambda}{\gamma-1} \cdot \frac{1}{V^{\gamma-1}}\Bigg|_{V_i}^{V_f}$$

$$= \frac{P_i V_i^{\gamma}}{\gamma-1}\left(\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right)$$

useful for fittings, plotting

can confirm $= C_v \Delta T$

$$W = \mathcal{E} = \frac{nRT_i}{V_i} \frac{V_i^{\gamma}}{\gamma - 1} \left(\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}} \right)$$

$$= \frac{nRT_i}{\gamma - 1} \cdot \left(\left(\frac{V_i}{V_f} \right)^{\gamma - 1} - 1 \right)$$

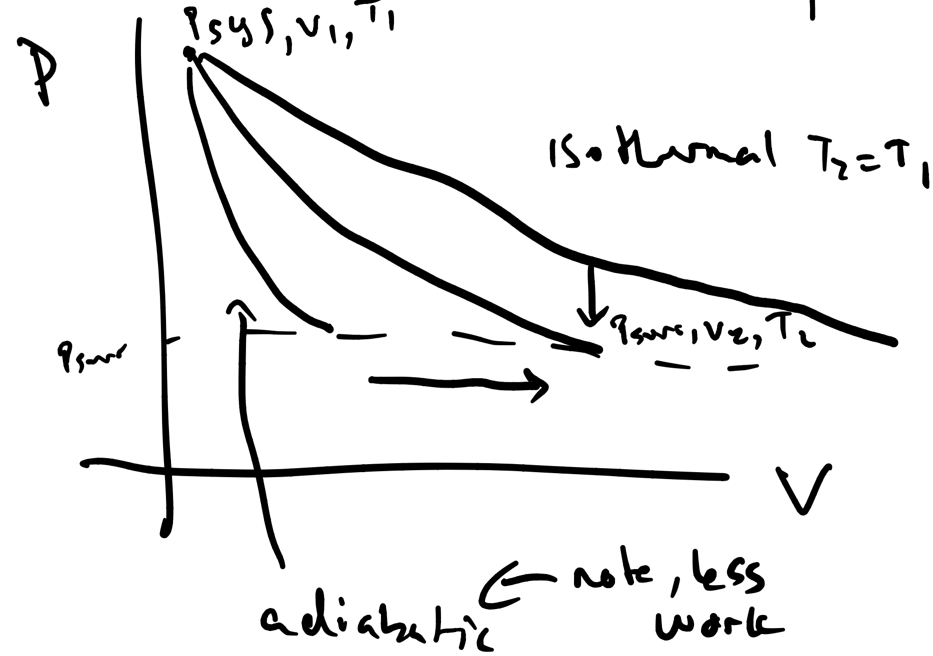
$$\gamma = \frac{C_v + nR}{C_v} \Rightarrow \gamma - 1 = \frac{C_v + nR}{C_v} - \frac{C_v}{C_v} = \frac{nR}{C_v}$$

$$= T_i C_v \left(\left(\frac{V_i}{V_f} \right)^{C_v / nR} - 1 \right)$$

$$= C_v (T_f - T_i) = C_v \Delta T$$

Irreversible expansion

pressure bath, $P_b < P_{sys}$



<u>adiabatic irreversible</u>

$w_{sys} = -w_{surr}$

$= P_{surr}(V_f^{surr} - V_i^{surr})$

$= -P_{surr}\Delta V_{sys}$

$\Delta U = -P_{surr}\Delta V_{sys}$

what is T final?

- Isothermal expansion followed by const vol cooling

(reaches eq. could also stop at V_f)

$$\Delta U = -P_{surr}\underbrace{\Delta V_{sys}}_{V_f - V_i} = C_V\Delta T = \underbrace{C_V}_{\frac{3}{2}nR}(T_f - T_1)$$

$$T_f = \frac{P_f V_f}{nR} \qquad T_i = \frac{P_i V_i}{nR}$$

$$-P_f V_f + P_f V_i = \frac{3}{2}(P_f V_f - P_i V_i) = \frac{3}{2}P_f V_f - \frac{3}{2}P_i V_i$$

$$P_f V_f = \frac{2}{5}\left[\frac{3}{2}P_i V_i + P_f V_i\right] \Rightarrow V_f = \frac{V_i}{5}\left(3\frac{P_i}{P_f} + 2\right) \checkmark$$

$$-P_f V_f + P_f V_i = \frac{3}{2} P_f V_f - \frac{3}{2} P_i V_i$$

$$\frac{3}{2} P_i V_i = \frac{5}{2} P_f V_f - P_f V_i$$

$$\Rightarrow P_f = \frac{3}{2} P_i V_i \cdot \frac{1}{\frac{5}{2} V_f - V_i}$$

$$= \frac{3 P_i V_i}{5 V_f - 2 V_i}$$

Expansion against vacuum

p121

Special case → $P_{surr} = 0$

work on surr $= -P_{surr} \Delta V = 0$

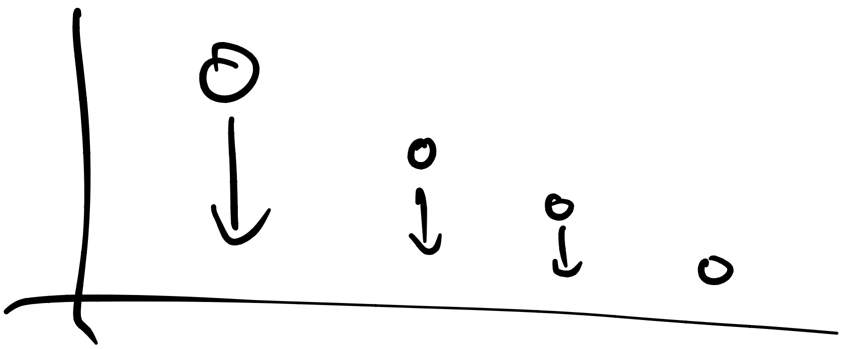
if adiabatic → $\Delta E = 0$, surprising!

but even 1 particle can push out plunger

Second law and entropy

Energy is conserved in all cases, so what is it that sets the direction of spontaneous processes

Eg. We don't see processes go backwards, such as



Energy of ball goes down, Where does it go?

Into heat / friction w/ ground & sound waves etc

What about reverse process?
Technically totally possible

Idea is energy spreads out, and this connects to what we already call entropy

Next time we will talk more about this spreading out idea

For now, will consider classical defn of entropy & where it comes from

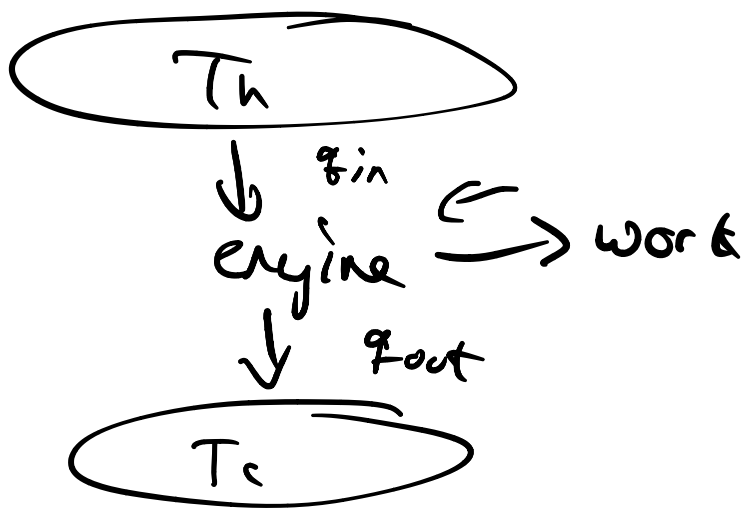
Original Clausius Statement:

2nd law: no process is possible where q goes from cold to hot

We will get more precise by defining entropy

__Engine__ A "cycle" (ends where it starts) that converts $\mathcal{E}$ from one form to another (mechanical work)

Diagram as

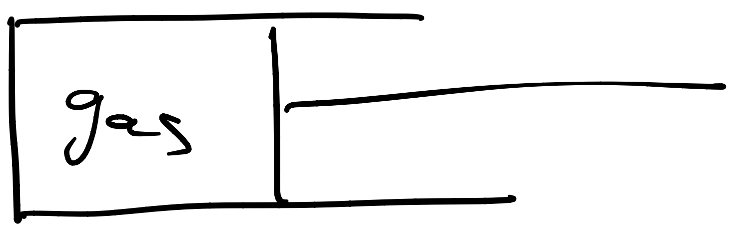


Not all received heat will convert to work (another 2nd law), so we can define an efficiancy

$$\epsilon = w/q_{in} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

$0 \leq \epsilon \leq 1$, will not get up to 1

Will do analysis for ideal engine – "Carnot cycle"



Cycle, expands at T_h then we push back in at T_c.

Net work comes from bath temp

4 steps All reversible

a) isothermal expansion @ T_h

connect to hot bath, release piston such that always stays at hot temp

q_{in}, w_{out}

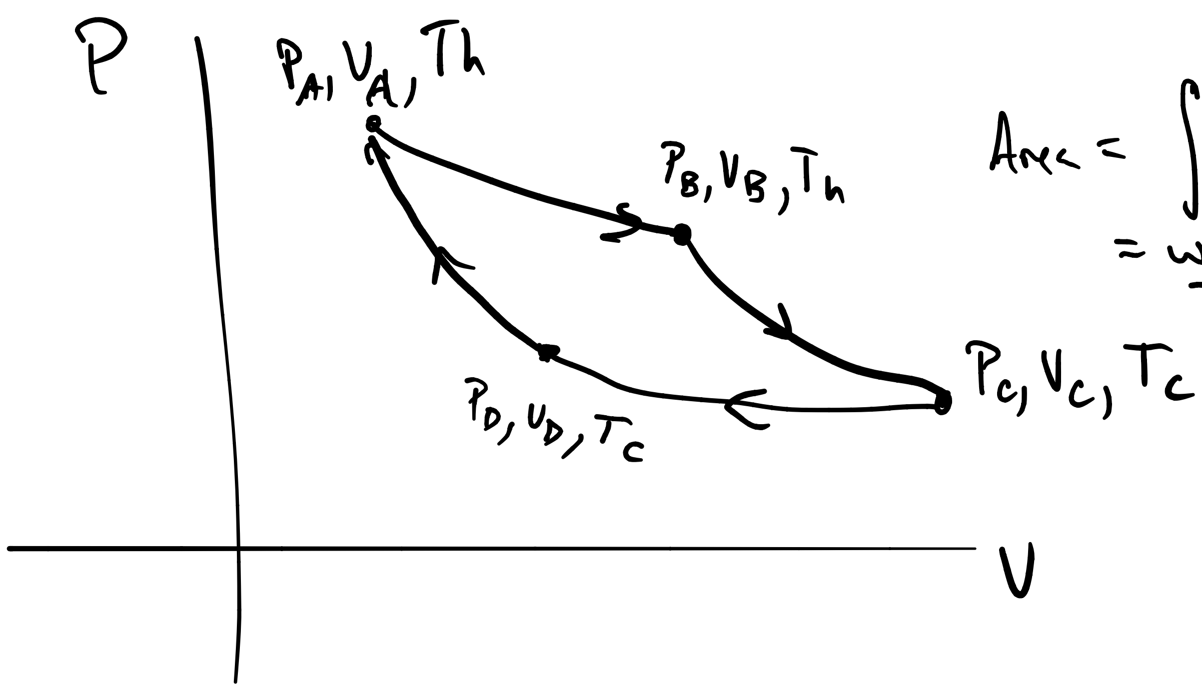
<u>Step 2</u> uncouple from bath, adiabatic
keeps expanding

$q = 0$, w_{out}

<u>Step 3</u> when system is at T_c, connect
to cold bath and push plunger in

$\underline{q_{out}}$, w_{in} Reversible isothermal compression

<u>Step 4</u> decouple & keep pushing in
plunger. T goes up, stop at T_h



Area $= \int P dV$
$=$ <u>work</u> <u>done</u>

Can see from graph, more work if $T_c \downarrow$ $T_h \uparrow$

this means extending adiabatic regions

what does it look like in T–V space?

Ideal gas helps us analyze heat & work properties

	flow in	work done
① isothermal expansion	$q = q_{in} = nRT\ln(V_B/V_A)$	$nRT\ln(V_B/V_A)$
	$\Delta E = 0,\quad q = -w$	
② adiabatic expansion	0 $q = 0$	$\Delta E = w = -C_V(T_c - T_h)$
③ isothermal compression	$q = -q_{out} = nRT\ln(V_D/V_C)$	$nRT\ln(V_D/V_C)$
④ adiabatic compression	0	$w = -C_V(T_h - T_c)$

For Cycle:

$$w_{tot} = w_1 + w_3 = nRT_h \ln(V_B/V_A) + nRT_c \ln(V_D/V_C) \leftarrow \text{neg}$$

Want to max T_h & min T_c