[Maybe no class on Dec 9, last class Dec 7 or 14]

Lecture 19

Eg $g(r)$ – relative probability of finding a particle distance r away from another particle

$$\langle A(r_1) B(r_2) \rangle$$ 2 point spatial correlation function

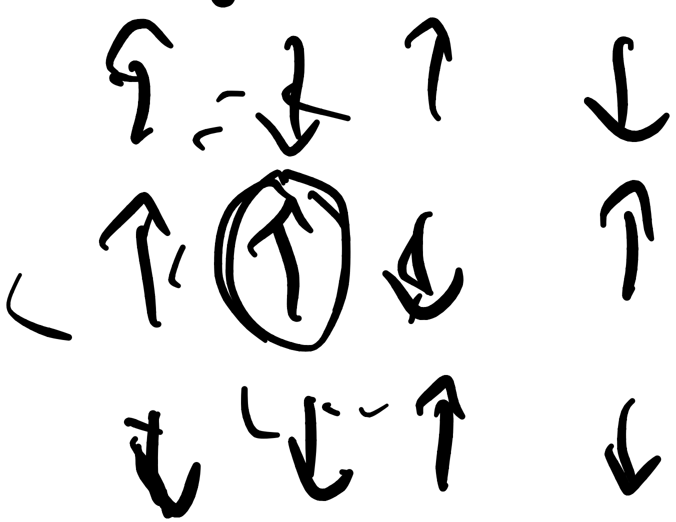
$$g(r) \sim \langle \rho(0)\,\rho(\vec{r})\rangle$$ ← density density

$$\rho(\vec{r}) = \sum_{i=1}^{N} \delta(\vec{r} - \vec{r}_i)$$ ← density

For magnets, spin-spin

$$\delta s_i = s_i - \langle s_i \rangle$$

$$C(r_{ij}) = \langle \delta s_i \, \delta s_j \rangle$$



"m" same

$$= \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

[uncorrelated ⇒ $\langle AB \rangle = \langle A \rangle \langle B \rangle$

as $r_{ij} \to \infty$ $\quad C(r_{ij}) \to 0$

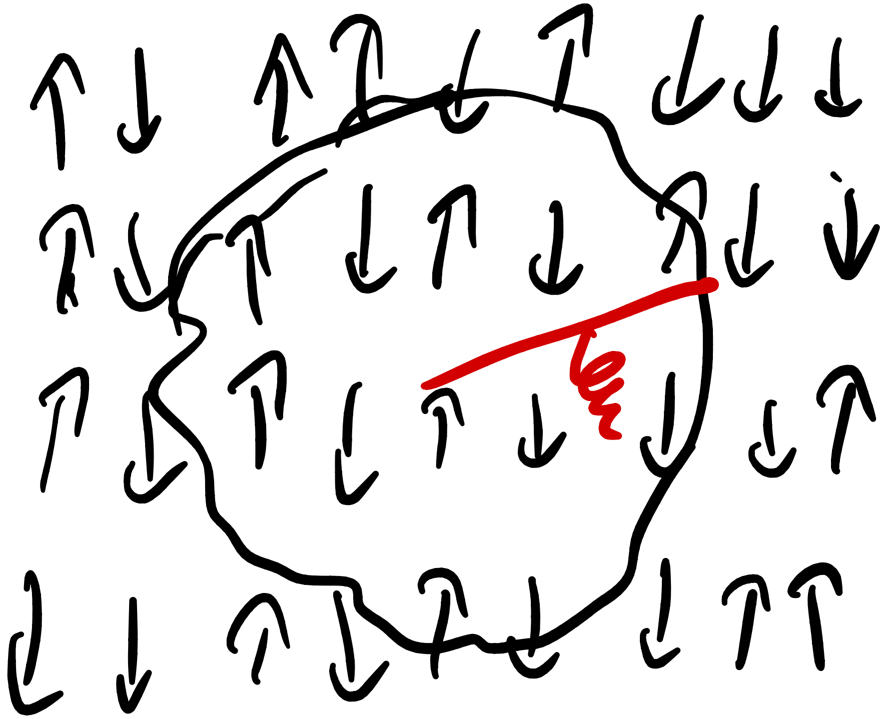
$$C(r_{ij}) = \langle s_i \quad s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

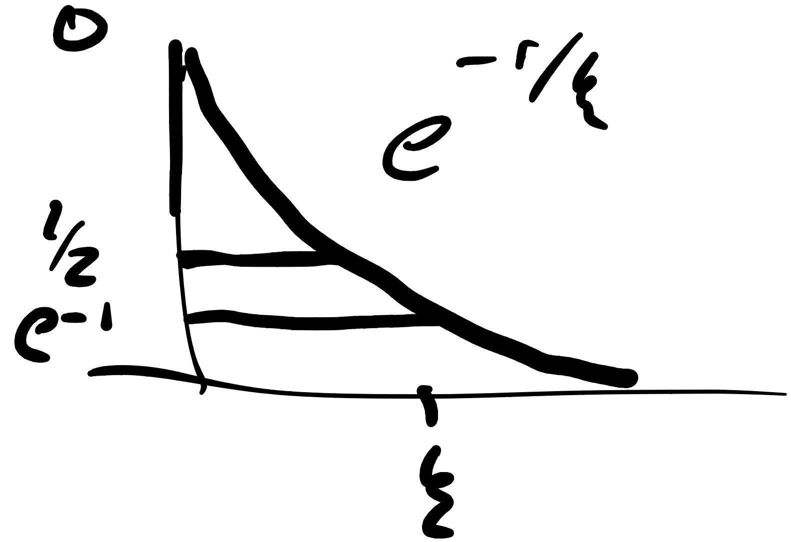
$$C(r_{ii}) = C(0) = \langle s_i^2 \rangle - \langle s_i \rangle^2$$

$$= \mathrm{Var}(s_i)$$

$$\tilde{C}(r_{ij}) = \frac{\langle \delta s_i \delta s_j \rangle}{\langle \delta s_i^2 \rangle}$$

$e^{-r/\xi}$





$$V_{\text{correlated}} \propto \xi^d \propto \#\text{ spins are correlated}$$

$$\#\text{ spins correlated w/ spin } i \sim \sum_j C(r_{ij})$$

$$\chi = \frac{1}{N} \sum_{ij} \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

$$\delta M = \sum_{i=1}^{N} s_i - \langle s_i \rangle$$

$$\chi \propto \langle (\delta M)^2 \rangle \propto \frac{\partial \langle m \rangle}{\partial h}$$

$$= \frac{N}{N} \sum_{j} \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

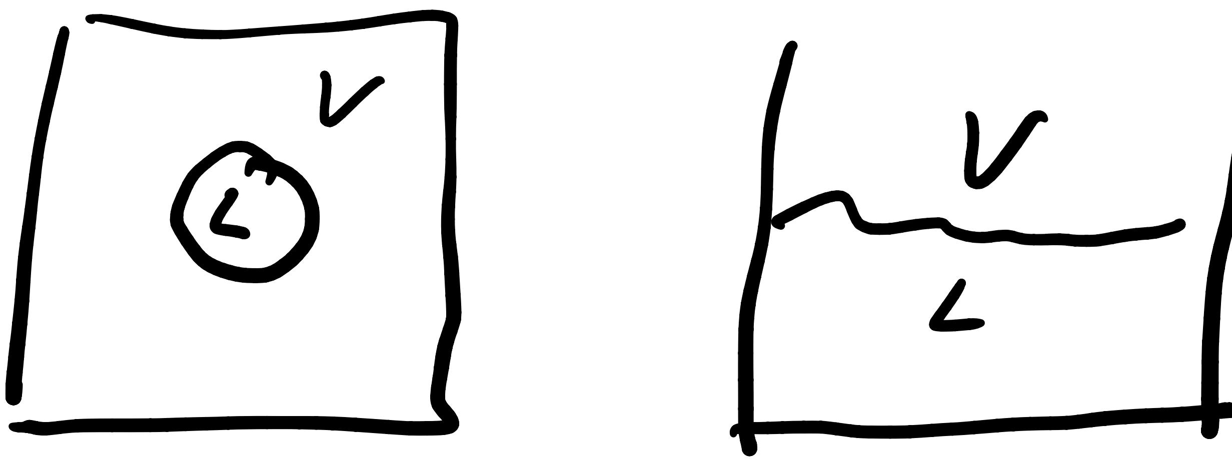
same for all i

$$= \sum_j C(r_{ij}) \propto \# \text{ spins correlated}$$

Susceptibility diverges at a phase transition

2 ways this can happen

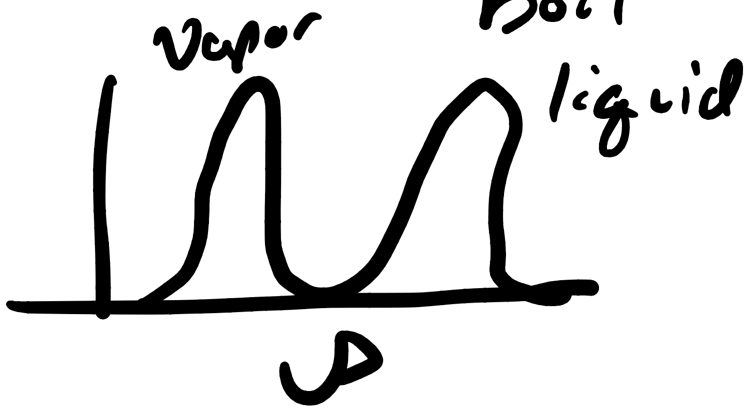
1) 1st order, phase coexistence



$\text{Var} \quad \int (\rho - \langle \rho \rangle)^2 P(\rho)$

at T_{boil}

$P(\rho)$



$\xrightarrow[\text{as } N \to \infty]{}$

$P(\rho)$

2) cts phase transition, 2nd order
no distinction between the phases

Susceptibility diverges at T_c

$$\chi \propto \xi^d$$, size of regions correlated $\to \infty$

$$C(r_{ij}) \sim \frac{e^{-r/\xi}}{r^{d-2+\eta}}$$

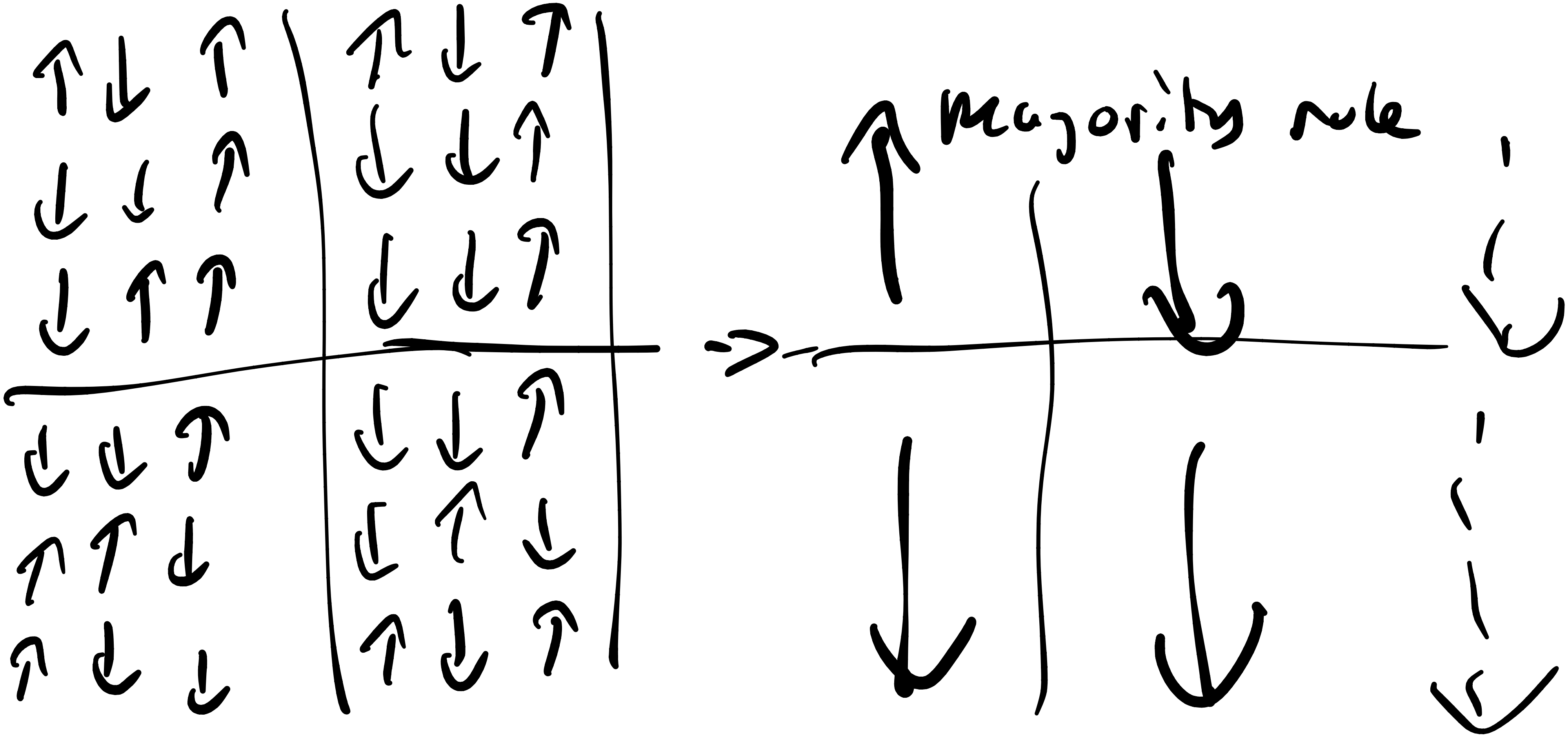
$T \to T_c$ $\xi \sim |T - T_c|^{-\nu}$

Renormalization
Group
Theory (RG)

Kadanoff

as you approach transitions
system looks "self similar"

[Ken Wilson]



Block spin renormalization

Want to be able to rewrite partition function so that "looks same"

$$Z = \sum_{\substack{\text{states} \\ 2^N}} e^{+\sum_{\langle ij \rangle} \beta J s_i s_j}$$

$\hookrightarrow$

1/9 spins

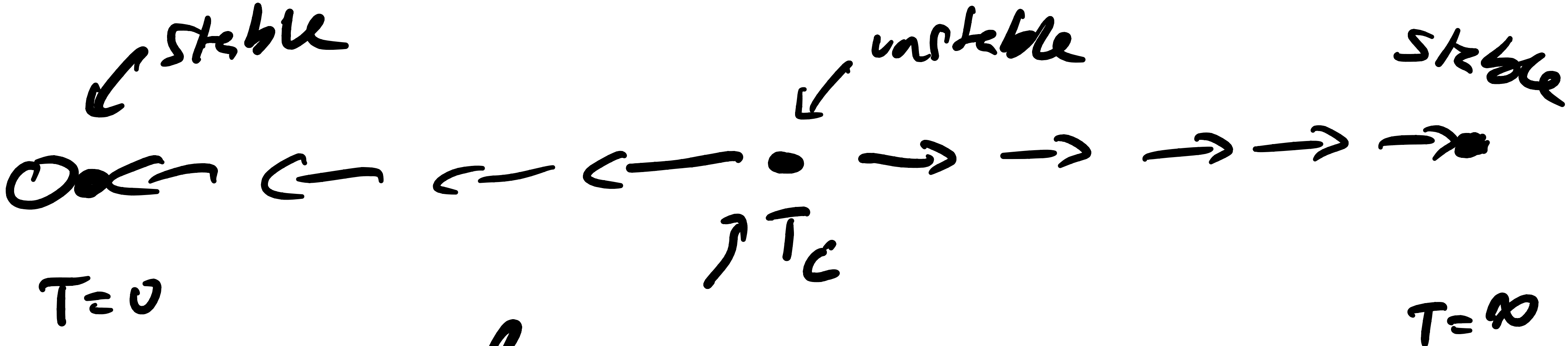
want

$$\sum_{\substack{\text{states} \\ 2^{(N/9)}}} e^{+\sum_{\langle ij \rangle} \beta J_2 \tilde{s}_i \tilde{s}_j}$$

not exactly possible

What can happen, $K_1 \to K_2 \to K_3 \to \dots$

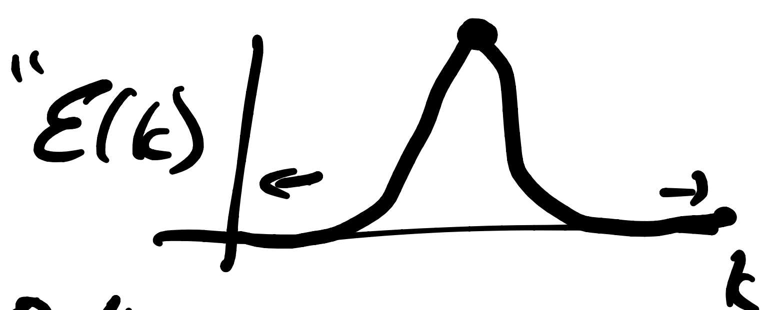
2d ising model



fixed points

[Book pg ~261, Chandler Ch 5]

- $J/k_B T_c = 0.50698$



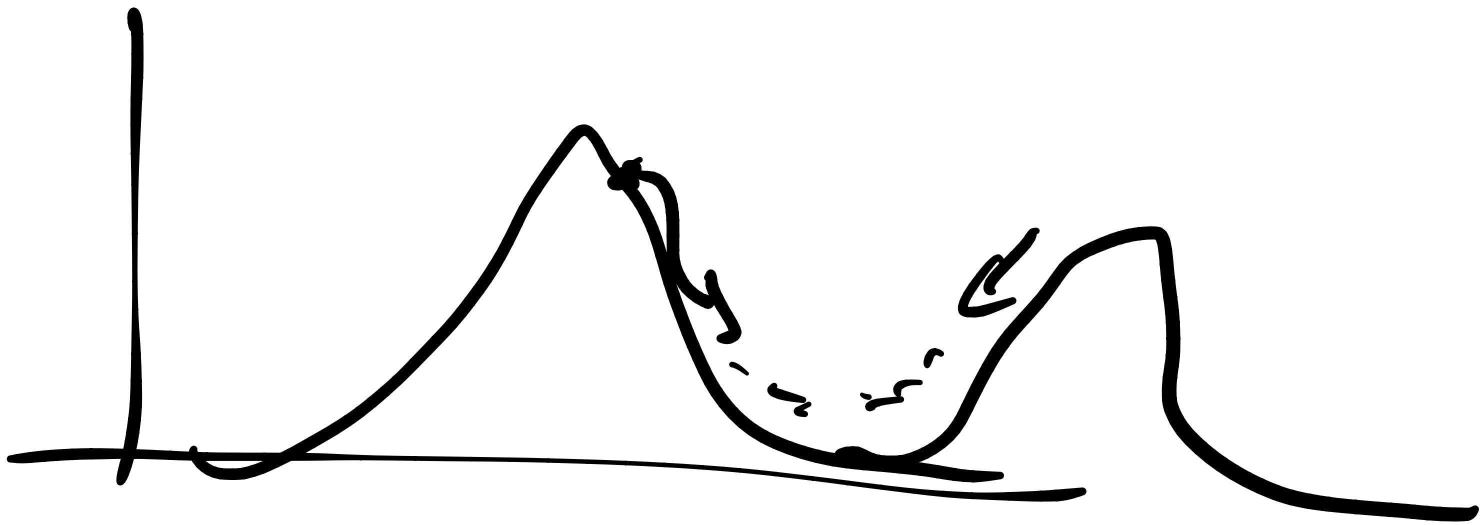
exact onsager $J/k_B T_c = 0.44069$

$\rightarrow \rightarrow \rightarrow \bullet \leftarrow \leftarrow \leftarrow$

T_{melt}

$Z(k) = 1 \quad \Rightarrow k = 11, \; Z = 4$

10 $\quad \Rightarrow k = 12, \; Z = 16$



Non-eq stat mech [R. Zwanzig]

Study of systems not at equilibrium

→ how systems reach equilibrium

→ what happens during this process

→ dynamics → • time reversibility can be broken

• "non-equilibrium steady state"

~ like equilibrium

out of equilibrium: "entropy production"

Eg: • self assembly by drying
in external field

• self propelled ~ active matter

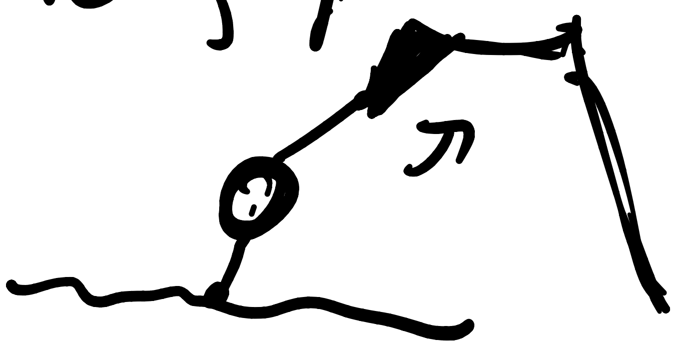
molecular motor

active
Janus particle

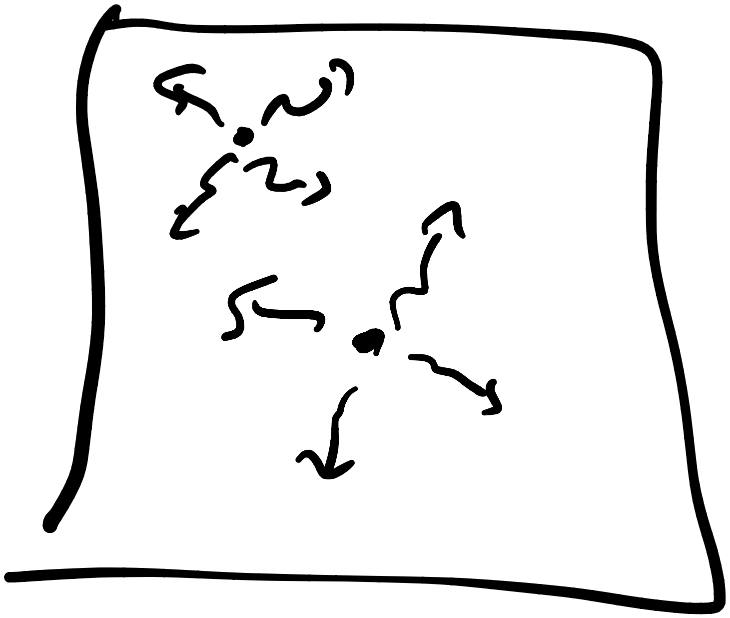
ATP → ADP
+ Pi

• (un)folding protein under force

Jarzynski



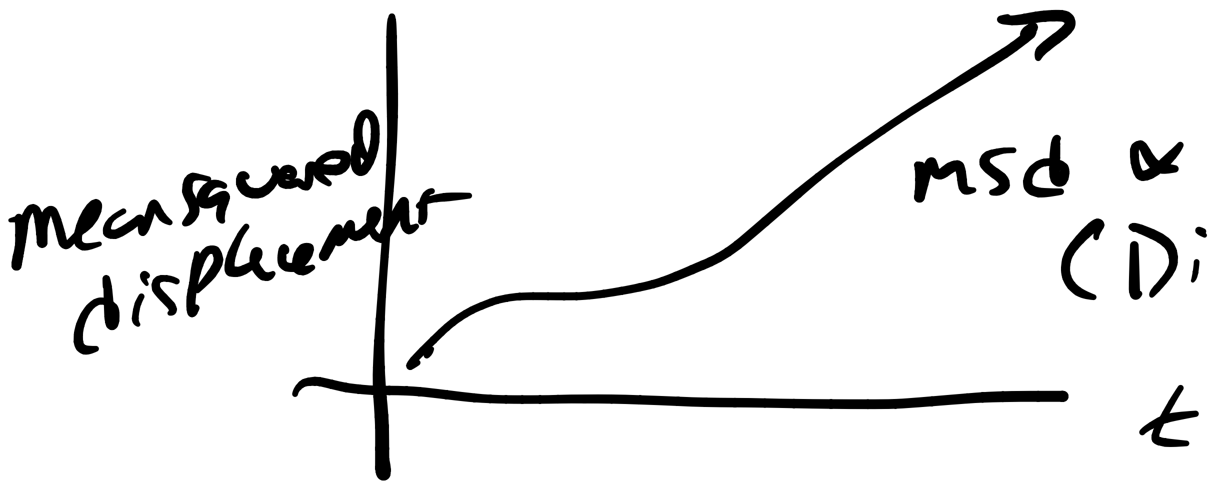
Brownian motion $\longleftrightarrow$ Langevin equation



Pollen particles

Einstein

$\rightarrow$ theory brownian motion



msd $\propto t$
(Diffusion)