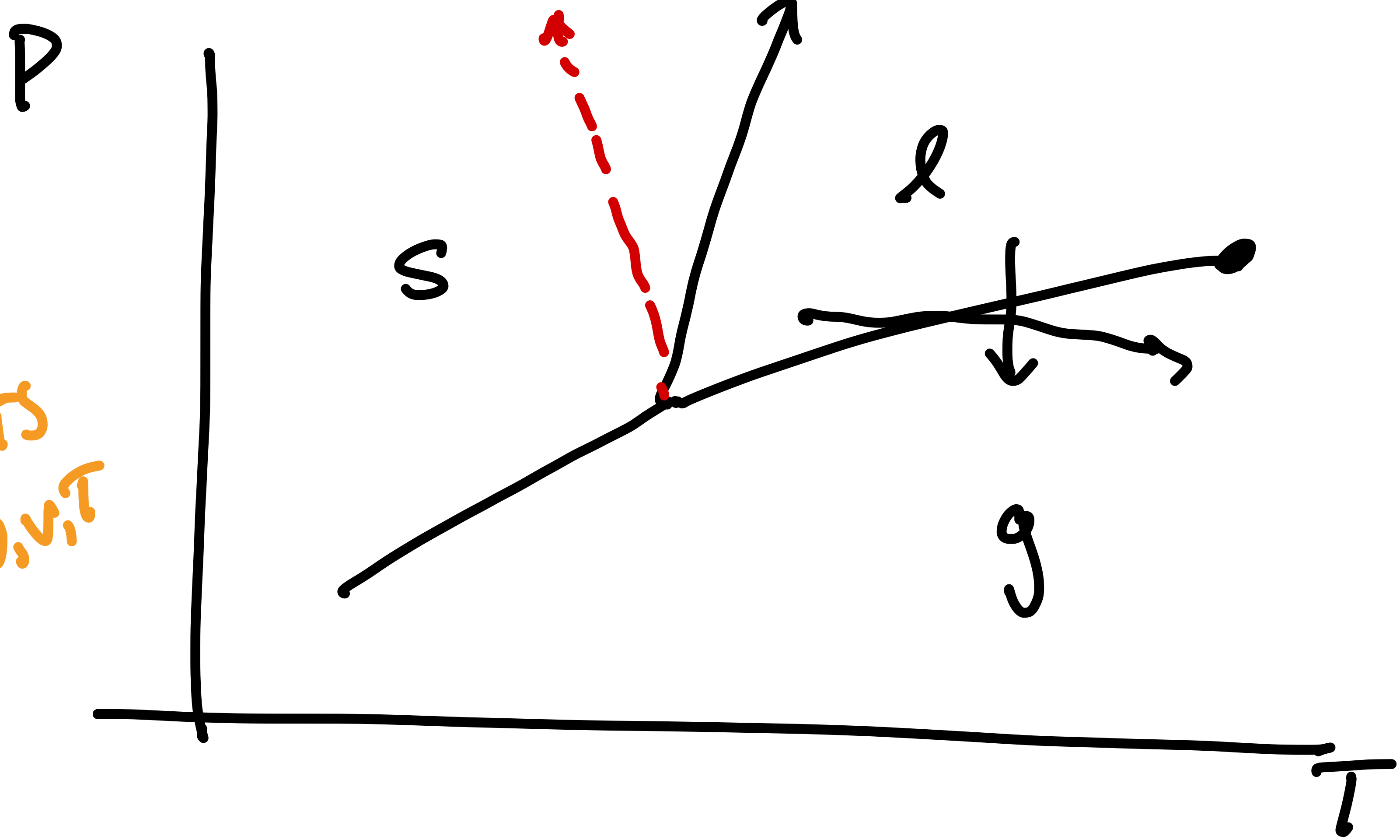
Lecture 16 - Phase transitions

Want to study:

1) What are the macroscopic property changes
[density, rigidity, color, viscosity]
2) Change is "microscopic" properties
crystal structure, dynamics

3) "Universality" - similarity between seemingly unlike systems

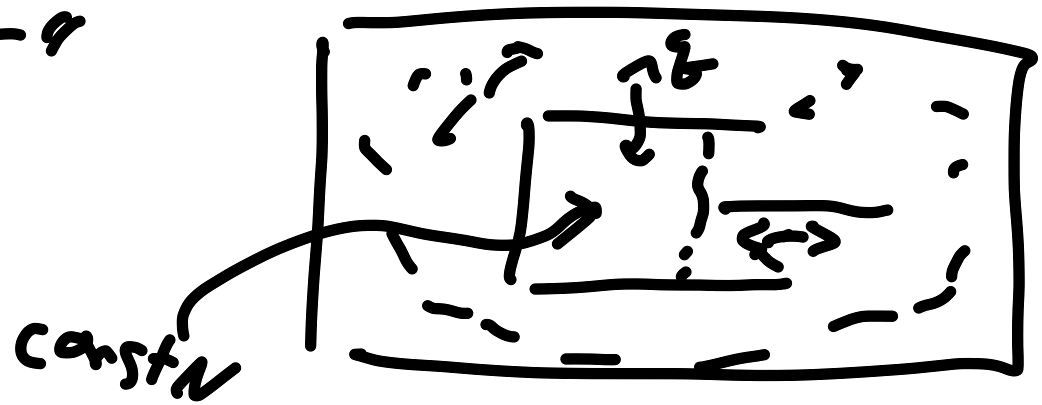


$A = \mathcal{E} - TS$

const N, V, T

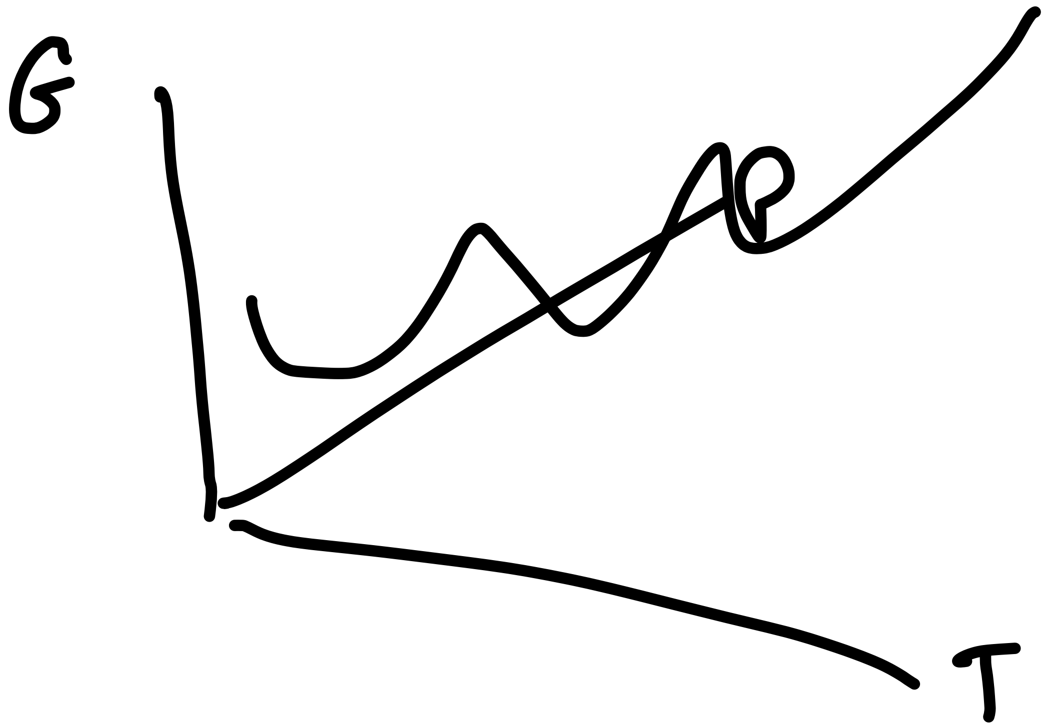
Isothermal - isobaric ensemble

"N, P, T"



$G(N, P, T)$
$= \mathcal{E} - TS + PV$

"Order parameter" ρ



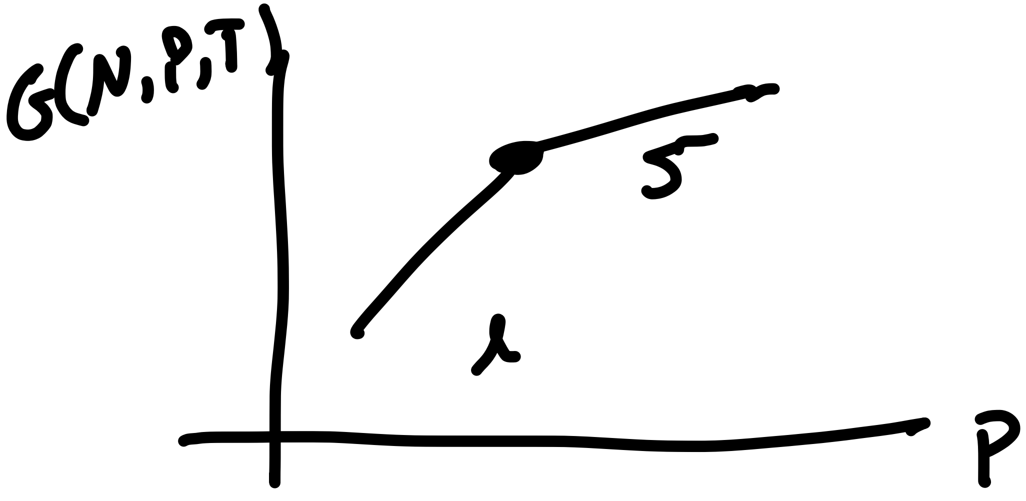
$P = 1$ atm

order parameter

L -> gas $\quad O(N,P,T) = \rho - \rho_L$

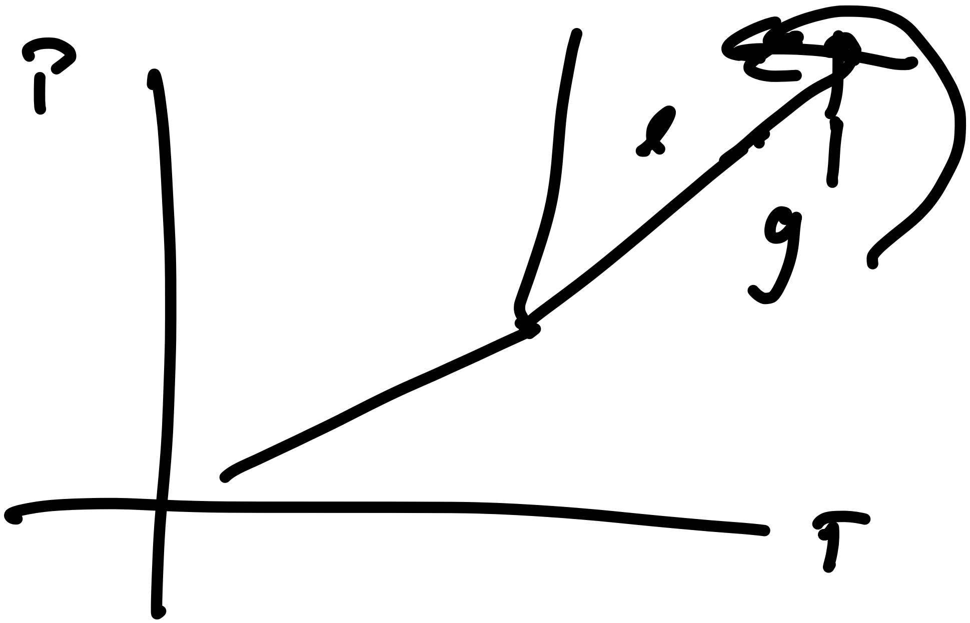
Cross a line: first order phase transition

Ehrenfest: discontinuity in first derivative of free energy



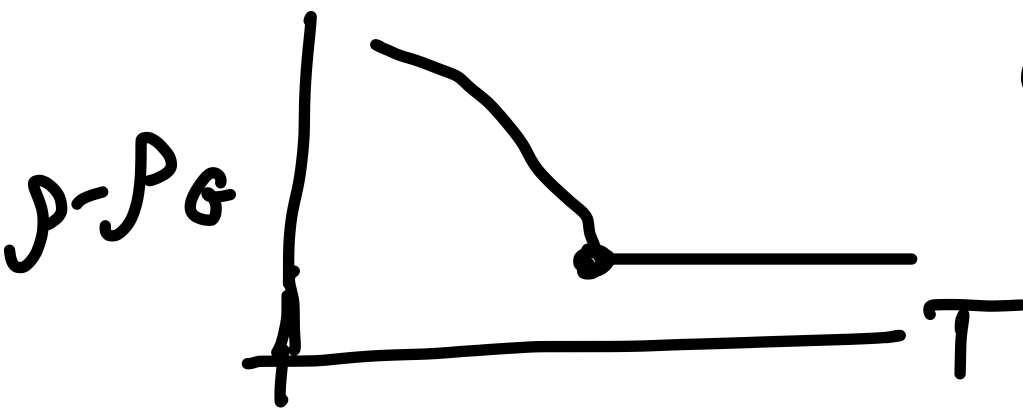
$V = \partial G / \partial P$

density $= N/V = N / (\partial G/\partial P)$



← no latent heat

2nd order phase transition (continuous)



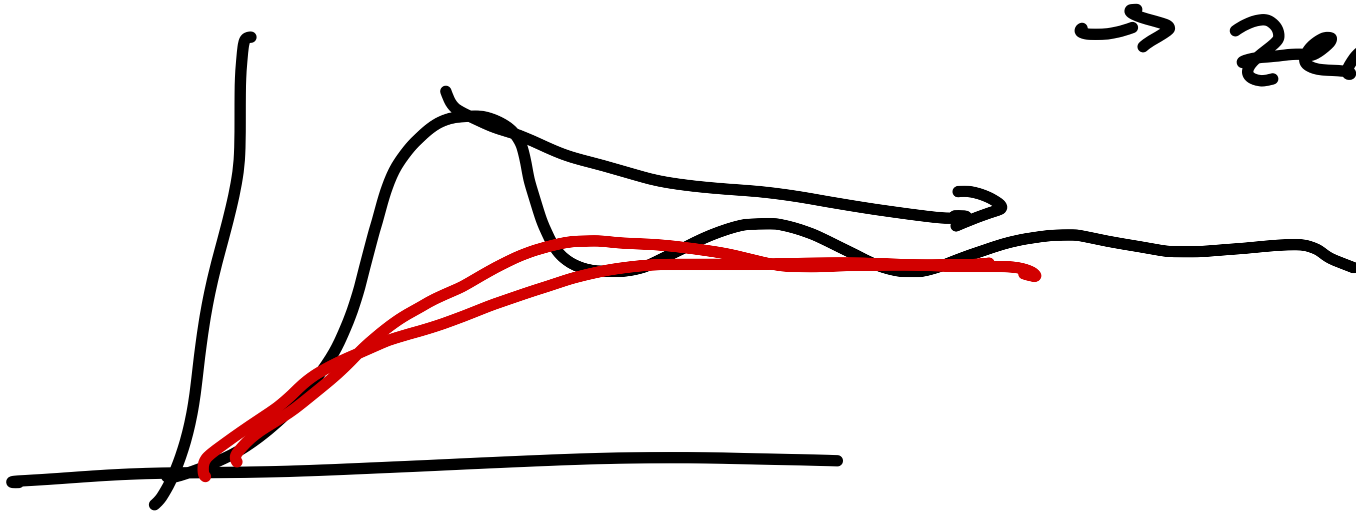
diverging susceptibility
diverging correlation length

Phase transitions

Break same kind of symmetry

Liq → solid   continuous → discrete "translational"

Liq → gas   finite correlation length → zero correlation length



Magnetization

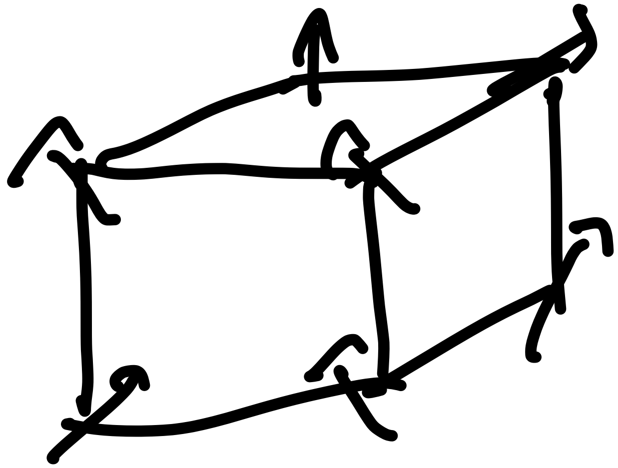
spins like to align

like to align w/ field

order parameter

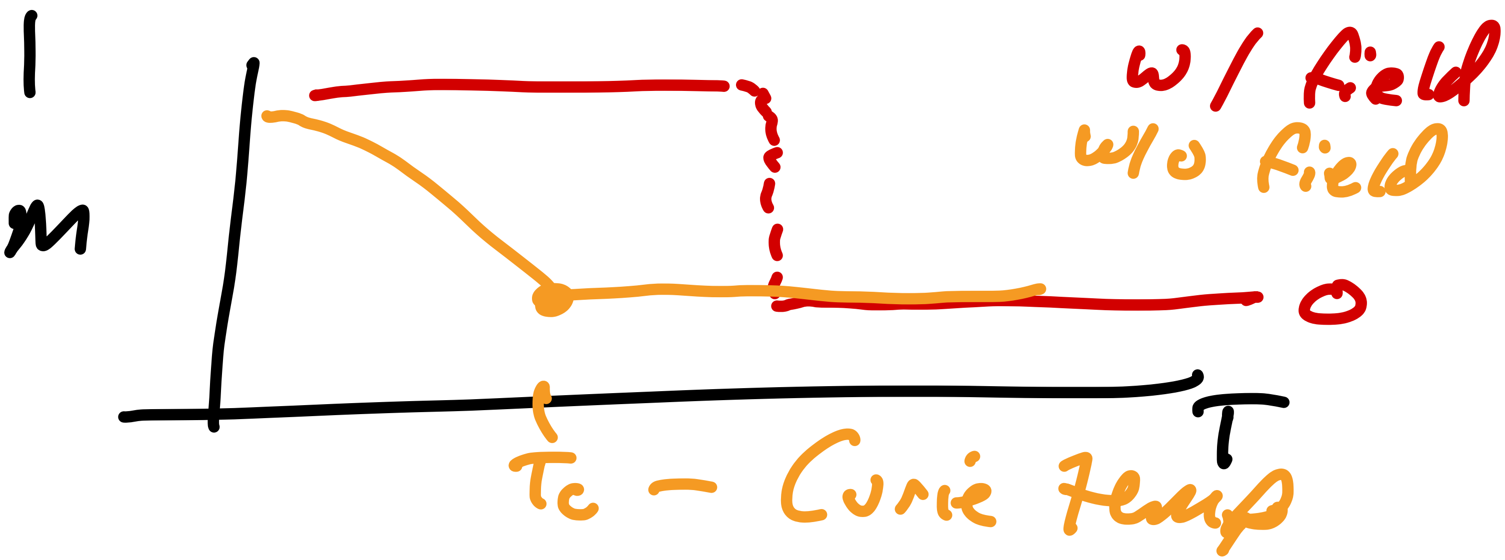
magnetization

N, V, T



$$A(N,V,T) = -k_B T \ln Z$$

$$M = \left| \left\langle \sum_{i=1}^{N} \sigma_i \right\rangle \right| , \quad m = M/N$$



$$\hat{\mathcal{H}} = -\sum_{ij} \hat{\sigma}_i \cdot J_{ij}\, \hat{\sigma}_j - \sum \gamma \vec{B} \cdot \hat{S}_i$$

$\hat{S}_i = \hbar/2\, \hat{\sigma}_i$ ← pauli matrix

↳ Ising model

neighbor coupling [# neigh $2 \cdot d$]

same coupling for all spins

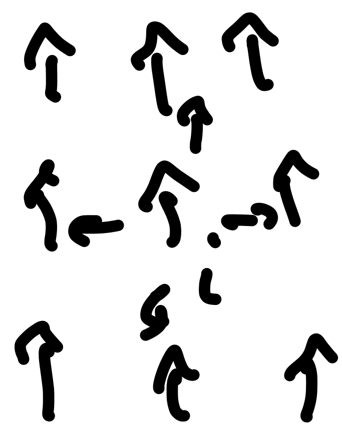
B field points up in $\hat{z}$

Ising

$$H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_i h s_i$$

sum over neigh → $\langle ij \rangle$

$s_i = \{\pm 1\}$



1 dimension:

no field

$\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

$m = 1$ $\qquad$ $\mathcal{E} = -NJ$

or

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$

$\cdots\ \uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\ \cdots$ $\qquad$ $m = 0$

$\mathcal{E} = -NJ + J$